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The Distributed Capacity of Inductance Coils. G. BREIT 649
Magnetic Susceptibility of Nickel and Cobalt Chloride Solutions. LAURA S. BRANT 678
The Occlusion of Water by Gas-mask Charcoal. BEN E. BROWN 700
The Luminescence of Certain Oxides sublimed in the Electric Arc. E. L. NICHOLS and D. T. WILBER 707
Errata 718
Index 719

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THE

PHYSICAL REVIEW.

THE DISTRIBUTED CAPACITY OF INDUCTANCE COILS.

By G. Breit.

Synopsis.

Effective Capacity of a Coil Defined.—Experiments show that if a coil is connected in series with a condenser of capacity C, the frequency $(\omega/2\pi)$ with which this combination is in resonance is given by $L(C + C_0) = 1/\omega^2$, where L and C_0 are constants.

The constant C_0 is called the *effective capacity* of the coil, sometimes simply "the capacity" of the coil. A general formula, equation (6), is derived for its calculation.

Single-layer Solenoid. The formula is applied to the short single-layer solenoid, used when grounded in an elliptical shield and when insulated from the shield, and to the short single-layer solenoid used when grounded and insulated in free space. An explanation is given of the remarkable *constancy of* C_0 as found by experiment in the case of short coils.

Experimental Verification.—An experimental verification is given by direct measurement of capacity and inductance. The current distribution in a coil has also been studied experimentally. The results have verified the theory.

Introduction.

THIS paper is intended to call the attention of physicists and mathematicians to some interesting aspects of the subject of distributed capacity of coils. The subject is of practical importance because inductance coils are used extensively in radio communication and because the distributed capacity, taken in connection with the value of inductance, determines the range of wave-lengths within which the inductance coil can be used to advantage. Furthermore, there is considerable mathematical interest connected with the calculation of the effective capacity caused by the capacity distributed along the wire of the coil.

The subject has been largely neglected by mathematical physicists. Lentz[1] and Drude[2] seem to be the only ones who have made a study of it.

[1] W. Lentz, Ann. d. Phys., *34*, p. 923–974, 1912. W. Lentz, Ann. d. Phys., *43*, p. 749–797, 1914.

[2] P. Drude, Ann. d. Phys., *9*, p. 293, 1902.

However, there are errors in Drude's mathematics, while Lentz's results are not adapted to numerical calculation and involve too many assumptions to be used generally. For example his treatment applies only to wires having circular section and further it combines the assumption of negligible curvature of a single turn with that of an infinitely long coil. This combination of assumptions may be justified for treatment of skin effect. Its applicability to capacity calculation is questionable.

In this paper an outline of the general method of calculating the effective capacity will be given and then illustrated by working out some special cases.

THE EFFECTIVE CAPACITY OF INDUCTANCE COILS.

Definition of Effective Capacity.—It is an experimental fact that the resonance frequency $\omega/2\pi$ of an inductance coil across whose terminals a condenser of capacity C is connected is given by

$$L(C + C_0) = \frac{1}{\omega^2} \tag{1}$$

where L, C_0 are constants for the coil in question. The international electrical units are used in this paper, where not otherwise specified. Formula (1) means that if an electromotive force $E_0 \cos \omega t$ acts in some part of the circuit and the capacity of the condenser is varied then the current is a maximum when (1) is satisfied. The constant C_0 is called the *Effective Capacity* of the coil. It is due to the capacities which are distributed along the wire of the coil.

The assumptions made in the following calculation are as follows:

1. The field of the condenser does not affect the field of the coil appreciably.

2. The resistance of the coil is negligible compared with its reactance, so that the wire may be treated as a perfect conductor.

3. The dimensions of the coil are so small in comparison with the wavelength used that the retarded values of the scalar and vector potential may be equated to their contemporaneous values.

4. The value of the E.M.F. induced in the whole coil by the current in a small section of the coil at a given point of the coil is proportional to the length of the section.

5. In the special cases considered the diameter of the coil is very much larger than the depth of the coil's winding or the axial length of the coil.

These assumptions will now be discussed in detail. Consider the case when the field of the condenser does not affect appreciably the electric intensity at the wires of the coil. It is essential to understand

that this may not always be the case as is seen from the following considerations.

The wire of the coil is connected to the condenser whose capacity is C. If there is a difference of potential between the two condenser plates the charges on the plates give rise to an electric field whose line integral along any curve between the two plates is equal to the difference of potential between the plates. If the curve is taken along the wire it follows that the electric intensity along all of the wire cannot be negligible since its line integral is equal to a large difference of potential.

Fig. 1.
Interleaving effect of condenser plates.

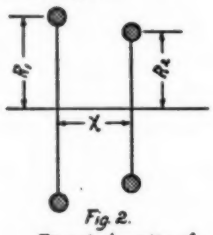


Fig. 2.
Diametral section of two turns of wire.

Fig. 3.
Cross section of multi-layer coil by a plane through the axis.

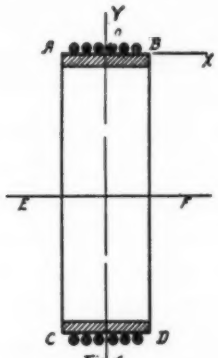


Fig. 4.
Diametral section of short single-layer solenoid.

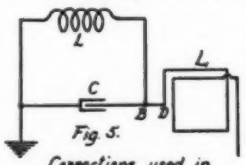


Fig. 5.
Connections used in measuring the capacity of a coil to ground.

Nevertheless it is legitimate to neglect this electric intensity under conditions which are satisfied in most practical cases. Consider the field which arises from the charges on the plates of an ordinary condenser whose plates are interleaved as shown on Fig. 1. The electric field is most intense between the plates. It also has a value at all points outside. But this value becomes small at a large distance. The principal contribution to the line integral is, therefore, confined to a small length along the leads connecting the coil to the condenser, and even if it should happen that there is an intense field due to the condenser at some part of the coil, it usually happens that this field is either compensated in some other part of the coil or else is along the wires of the coil.

However, there are cases when this assumption cannot be justified. No general treatment of these cases can be given unless the dimensions

and position of the condenser are known. For this reason the present treatment considers the ideal case when the condenser is connected to the coil by long leads and in practical applications one must constantly bear in mind that this assumption has been made.

Let, then, the electric intensity caused by the condenser be negligible and let the electromotive force be applied between the condenser and the coil in one of the long leads. Then the electric intensity at any point on the surface of the coil is due solely to two causes:

(1) the currents in the coil,

(2) the charges on the wire of the coil.

If further the material out of which the coil is made should be a perfect conductor then the electric intensity must be normal to the surface of the coil at every point.

The question arises immediately as to whether the current is the same through every cross section of the wire. It is apparent at once that it is not. In fact the current in the coil gives rise to an electric intensity which has a component tangential to the wire at least at some points because if this were not the case the coil would not have any self-inductance, because the self-inductance is the line integral along the wire of the electric field due to the current per unit rate of change of current. But the total electric intensity must be normal to the surface of the wire and therefore has no tangential component. Consequently there must be charges on some parts of the coil which give rise to an electric intensity whose tangential component is equal and opposite to the tangential component due to the current. In the case considered the tangential component of the current varies periodically. Therefore, the charges also vary periodically. If x is an arbitrary parameter along the wire such, *e.g.*, as a length measured from an arbitrary fixed point along the wire, if i is the current, and if Qdx is the net charge in an infinitesimal segment dx, *i.e.*, if Q is the charge per unit length, then it is easily shown that

$$\frac{\partial i}{\partial x} = -\frac{\partial Q}{\partial t} \tag{2}$$

by the conservation of charge. As shown

$$\frac{\partial Q}{\partial t} \neq 0.$$

Therefore, $\partial i/\partial x \neq 0$. Thus the current is different at different points on the wire.

Suppose that the frequency dealt with is so low that this non-uniformity of current is small. Then as an approximation the component of the

electric intensity may be computed as if a uniform current i were flowing through the coil. Let (1) and (2) be arbitrarily selected points on the surface of the wire of the coil. Let the self-inductance of the portion of the coil included between (1) and (2) be L_{12} and denote the mutual inductance of this section to the rest of the coil by M_{12}. Then the line integral of the electric intensity due to i and taken along the wire from (1) to (2), i being reckoned positive when the current is flowing from (1) to (2), is

$$e_{12} = (L_{12} + M_{12}) \frac{\partial i}{\partial t}.$$

If the electrostatic potential is V_1 at (1) and V_2 at (2), the line integral of the electric intensity due to the charges and taken from (1) to (2) is

$$V_1 - V_2.$$

The total line integral is then

$$e_{12} + V_1 - V_2.$$

But the total electric intensity is normal to the wire and, therefore, the total line integral is zero. Hence

$$V_2 - V_1 = e_{12}, \tag{3}$$

i.e., the line integral of the field due to the current between any two points on the surface of the wire is equal to the difference of potential between the two points.

Attention must be called here to the fact that e_{12} is independent of the path on the surface of the wire along which the integration is effected because no matter how intricate this path may be the result of the integration must always give $V_2 - V_1$. This means that even though the electric field due to the currents cannot be derived from a single-valued potential at all points in space, the component tangential to the surface of a perfect conductor may be derived from a single-valued potential on the surface of that conductor.

The same may also be seen from the fact that the magnetic intensity has a constant normal component to the surface of a perfect conductor so that the flux of magnetic induction through any closed curve on the surface of the conductor is constant and the line integral of the electric intensity around any closed curve is zero.

Thus from a knowledge of i, $V_2 - V_1$ may be derived. If it is also known that some part of the coil is grounded then the point grounded is at zero potential and therefore V is known at all points of the coil. According to the assumption made as to the condenser the only charges which give rise to V are those on the wire of the coil and the objects in

the neighborhood of the coil. Suppose that it is possible to solve the electrostatic problem of finding such a distribution of charges on the coil and its surroundings as to give rise to the specified value of V. This problem can be always solved and, moreover, the solution of it is unique because it is equivalent to finding V in a region of space within which V satisfies Laplace's equation and on the boundaries of which V is known. (Theoretically it is known that such a distribution and that only one such distribution exists.) Then Q in (2) is known and consequently i may be obtained from the formula

$$i = i_1 - \int_{x_1}^{x} \frac{\partial Q}{\partial t} dx,$$

where i_1 is the value of i at $x = x_1$.

Further, since the relation between the charge density $Q(x)$ and the "difference of potential" between the coil terminals is necessarily linear one can conveniently write

$$Q(x) = + \alpha(x) \frac{di_1}{dt}, \tag{4}$$

where $\alpha(x)$ is a function of x whose form depends on the shape and dimensions of the coil and the nature of the medium around the coil. The expression di/dt is written only as an approximation, which, however, is legitimate if $i - i_1$ is small because in the computation carried out only the terms depending on the first power of the frequency will be taken into account and $Q(x)$ itself depends on the first power and powers higher than the first. Since $i - i_1$ is zero for direct current, it is small for a sufficiently low frequency. Thus

$$i(x) = i_1 - \left(\frac{d^2i}{dt^2}\right) \int_{x_1}^{x} \alpha(x) dx, \tag{5}$$

where x_1 is the value of x at one of the terminals of the coil.

The coil may be divided mentally into a number of small sections. Thus if the coil has 100 turns each of the turns may be looked at as one such small section. Within one of the sections the current is uniform. If the parameter assigned to one of them is x, the current in the section is $i(x)$. Then $i(x)$ is given by the preceding equation (5). If the section is small enough it is possible to find such a function $M(x)$ that $M(x)\Delta x$ is the sum of the self-inductance and the mutual inductance to the rest of the soil for a section included between x and $x + \Delta x$. The electromotive force induced between the coil terminals due to the current in the section is then

$$M(x) \frac{\partial i(x)}{\partial t} \Delta x.$$

The electromotive force due to all sections is

$$e_{12} = -\Sigma M(x)\frac{\partial i(x)}{\partial t}\Delta x,$$

where the summation is extended over all the sections. If Δx becomes infinitesimal this summation is

$$e_{12} = -\int_{x_1}^{x_2} M(x)\frac{\partial i(x)}{\partial t}dx$$

where x_1, x_2 are the values of x at the ends of the coil. Substituting the value of $i(x)$ given by (5) the electromotive force is:

$$e_{12} = -\frac{di_1}{dt}L + \frac{d^3i_1}{dt^3}\int_{x_1}^{x_2} M(x)\left\{\int_{x_1}^{x}\alpha(x)dx\right\}dx,$$

where

$$L = \int_{x_1}^{x_2} M(x)dx$$

and is therefore the self-inductance of the coil. If the oscillations in the current i are simple harmonic, *i.e.*, if they are represented by

$$i = I_0\cos(\omega t - \theta),$$

then

$$\frac{d^3i_1}{dt^3} = -\omega^2\frac{di_1}{dt},$$

so that the electromotive force induced in the whole coil is

$$e_{12} = -L\frac{di_1}{dt}\left[1 + \omega^2\int_{x_1}^{x_2}\frac{M(x)}{L}\left\{\int_{x_1}^{x}\alpha(x)dx\right\}dx\right].$$

The current i_1 is the current which flows through the condenser connected to the coil to constitute an oscillatory circuit. Let C be the capacity of the condenser and $-Q_c$ the charge on the plate connected to the ungrounded point of the coil given by $x = x_1$. Then

$$\frac{dQ_c}{dt} = i_1$$

and

$$\frac{di_1}{dt} = -\omega^2 Q_c.$$

Therefore, in terms of Q_c the electromotive force is

$$e_{12} = L\omega^2 Q_c\left[1 + \omega^2\int_{x_1}^{x_2}\frac{M(x)}{L}\left\{\int_{x_1}^{x}\alpha(x)dx\right\}dx\right].$$

But the difference between the potential of the plate connected to $x = x_2$

and the plate connected to $x = x_1$ is Q_c/C. Therefore,

$$\frac{Q_c}{C} = Q_c L\omega^2 \left[1 + \omega^2 \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \alpha(x)dx \right\} dx \right].$$

If

$$\omega^2 \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \alpha(x)dx \right\} dx$$

is small compared to 1, that is, if ω is sufficiently small, the above formula gives

$$\omega^2 = \frac{1}{LC}.$$

Using this value of ω^2 and substituting in the integral term of the above expression a second approximation is obtained in the form

$$\frac{1}{\omega^2} = L\left[C + \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \frac{\alpha(x)}{L} dx \right\} dx \right].$$

If this is to be identical with

$$\frac{1}{\omega^2} = L(C + C_0)$$

which is the experimental relation then

$$C_0 = \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \frac{\alpha(x)}{L} dx \right\} dx. \tag{6}$$

Although approximations were made in the formula it is general and correct as long as C_0 is a constant because the formula is exact at low frequencies. For convenience of reference the meaning of the symbols in this formula is restated.

If the charge in an element dx is $Q(x)dx$, then

$$\alpha(x) = \frac{Q(x)}{\left(\frac{di}{dt}\right)},$$

$M(x)dx$ is the mutual inductance between the section dx and the whole coil. L is the self-inductance. x is an arbitrary parameter. x_1, x_2 are the terminal values of x, x_1 being the value at the ungrounded condenser terminal.

The main steps in the derivation of this formula are summarized below:

(*a*) Since the wire was assumed to be a perfect conductor the E.M.F. induced between any two points is equal to the difference of potential between the points on the surface.

(*b*) This difference of potential calls for charges on the surface of the wire.

(*c*) The charges on the wire cause a non-uniform distribution of current in the wire.

(*d*) When the frequency is low the E.M.F. induced between any two points may be calculated as if the current were uniform. Hence the potential at all points of the wire is known and, therefore, the charge in any section is known.

(*e*) Knowing the distribution of charge the distribution of current is derived.

(*f*) From a knowledge of the connection between the current at the ungrounded condenser terminal and the current at any other point the self-induced E.M.F. is expressed in terms of the current at the condenser terminal.

(*g*) This E.M.F. is equated to the potential difference between the condenser plates expressed in terms of the charge and capacity. By comparison with the formula

$$L(C + C_0) = \frac{\mathrm{I}}{\omega^2}$$

C_0 is derived.

(*h*) The theory given so far applies strictly only to very low frequencies. However, it will be shown later that C_0 is independent of the frequency for many cases. This is true experimentally in all important cases. Therefore, in the cases when C_0 does not vary with the frequency the formula (6) is general.

Applications to Problems Solved by Two-Dimensional Methods.

In this paper only problems solved by two-dimensional methods will be considered. A large class of problems reduce themselves to two-dimensional problems. This is the case with any circular coil whose diameter is large compared to the maximum distance between two of its turns.

Let R_1 be the radius of one of the turns. The charge density per unit length of wire is sensibly constant within one turn, if the number of turns is large. Let this charge density be ρ_1. The potential due to the turn considered at any point in space is now obtained by direct integration. By symmetry it is the same at all points of any circle coaxial with the first. Let this circle be of radius R_2, and let x denote the distance between the planes of the two circles. (See Fig. 2.) Then the potential in volts is

$$V = \frac{c^2}{10^9 K} \int_0^{2\pi} \frac{\rho_1 R_1 d\varphi}{\sqrt{(R_1 \cos \varphi - R_2)^2 + R_1^2 \sin^2 \varphi + x^2}},$$

where $c = 2.9982 \times 10^{10}$ and is the ratio of the electromagnetic to the

electrostatic unit of charge. K is the specific inductive capacity of the medium around the coil. Now

$$\int_0^{2\pi} \frac{d\varphi}{\sqrt{(R_1\cos\varphi - R_2)^2 + R_1^2\sin^2\varphi + x^2}}$$

$$= 4\int_0^{\pi/2} \frac{d\varphi}{\sqrt{x^2 + (R_1+R_2)^2 - 4R_1R_2\cos^2\varphi}}$$

$$= \frac{4}{\sqrt{x^2+(R_1+R_2)^2}} F\left(\frac{2\sqrt{R_1R_2}}{\sqrt{x^2+(R_1+R_2)^2}}\right),$$

where $F(k)$ denotes complete elliptic integral of the first kind with modulus k.

If x is small compared to both R_1 and R_2

$$\frac{2\sqrt{R_1R_2}}{\sqrt{(R_1+R_2)^2+x^2}}$$

is nearly 1. Consequently the elliptic integral

$$F\left(\frac{2\sqrt{R_1R_2}}{\sqrt{(R_1+R_2)^2+x^2}}\right)$$

is approximately[1]

[1] See Appendix

$$\log \frac{4}{\sqrt{1 - \dfrac{4R_1R_2}{(R_1+R_2)^2+x^2}}} = \log\left(\frac{8R}{d}\right),$$

where

$$d = \sqrt{(R_1-R_2)^2+x^2}$$

and where

$$\sqrt{(R_1+R_2)^2+x^2}$$

is replaced by $2R$, R now being used for R_1 as well as R_2. Thus

$$V = \frac{2\times 10^{-9}c^2}{K}\rho_1 \log\left(\frac{8R}{d}\right) = \frac{17.978\times 10^{11}}{K}\rho_1\log\left(\frac{8R}{d}\right). \qquad (7)$$

This is the approximate expression for the potential caused by a circular ring at a small distance from it. It is essential to note here that the potential caused by the same ring at infinity vanishes to the first order.

Let then $ABCD$ (see Fig. 3) represent a section of the coil by a plane through its axis, the small circles being the sections of the wires of the coil. Each one of the wires is charged, the charge being distributed with a certain surface density, say σ. Let ds be an element of length along one of the circles. Then $\sigma\, ds$ is the corresponding linear density of charge for a ring of width ds and on the surface of the wire. If r is

the distance from any point in the cross section to ds, then the potential at that point is obtained as

$$V = \Sigma \frac{10^{-9}c^2}{K} \int 2\sigma \log\left(\frac{8R}{r}\right) ds = \Sigma \frac{8.989 \times 10^{11}}{K} \int 2\sigma \log\left(\frac{8R}{r}\right) ds,$$

where the integration extends over the whole circle and the summation covers all the circles. V can be broken up into two parts by writing

$$V = -\Sigma \frac{8.989 \times 10^{11}}{K} \int 2\sigma \log (r) ds + (\log 8R) \Sigma \frac{8.989 \times 10^{11}}{K} 2\sigma ds.$$

The second of these two vanishes if

$$\Sigma \int \sigma ds = 0,$$

i.e., if the total charge on the wires of the coil remains constant. Such is the cause if the coil is not grounded. In this case of the ungrounded coil the first term is the only term and consequently the potential is the same as that which would exist if the wires of the coil were straight because a straight wire may be considered as the limiting case of a circular wire of infinite radius. *Thus the case of the ungrounded short coil is always reducible to a two-dimensional problem.*

If the short coil is grounded the only modification is that introduced by the second term. This contributes the same potential at all points of the cross section. Thus only a slight modification in the first solution is introduced by grounding. It must be carefully remembered here that in the two-dimensional case the logarithmic potential is infinite both at the filament causing it and at infinity. This makes it impossible to apply the two-dimensional treatment in general. If, however, the total charge is zero, the potential vanishes at infinity in the two-dimensional case and as was shown above is approximately the same as in the case of circular turns. It is only in this case, therefore, that the two-dimensional treatment applies.

There is an additional simplification in the case considered. This is introduced by the fact that *the E.M.F. induced in any one turn is practically the same as that in any other.*

In fact the expression for the mutual inductance between two coaxial circles whose radius R is large compared to their distance apart d is

$$M = 4\pi R \left[\log\left(\frac{8R}{d}\right) - 2 \right].$$

Draw a system of rectangular axes (OX, OY) in the plane perpendicular to the wires of the coil. To each circle coaxial with the coil there corresponds a point (x, y), the point of intersection of the circle with the plane.

Suppose the current distribution is given by the fact that a current

$$\frac{1}{\mu} f(x, y)dxdy$$

crosses an area $dx\,dy$ at (x, y), μ being here the permeability of the medium. Then the flux through a circle passing through the point (x_0, y_0) is

$$N = 4\pi R \int\int \left\{ \log \left(\frac{8R}{\epsilon^2 \sqrt{(x - x_0)^2 + (y - y_0)^2}} \right) \right\} f(x, y)dxdy,$$

where the double integral is taken over the entire area through which a current may pass, and ϵ is the natural base. Let a be the maximum possible value of

$$\sqrt{(x - x_0)^2 + (y - y_0)^2}.$$

Then

$$\log \frac{a}{\sqrt{(x - x_0)^2 + (y - y_0)^2}}$$

is always either zero or positive.

But

$$N = 4\pi R \int\int \left\{ \log \left(\frac{8R}{a\epsilon^2} \right) \right\} f(x, y)dxdy + 4\pi R \int\int \left\{ \log \frac{a}{\sqrt{(x - x_0)^2 + (y - y_0)^2}} \right\} f(x, y)dxdy.$$

Therefore,

$$N > 4\pi R \left\{ \log \left(\frac{8R}{a\epsilon^2} \right) \right\} \int\int f(x, y)dxdy. \tag{8}$$

Further,

$$\begin{aligned} \frac{\partial N}{\partial x_0} &= 4\pi R \int\int \frac{x_0 - x}{(x_0 - x)^2 + (y_0 - y)^2} dxdy, \\ \frac{\partial N}{\partial y_0} &= 4\pi R \int\int \frac{y_0 - y}{(x_0 - x)^2 + (y_0 - y)^2} dxdy, \end{aligned} \tag{9}$$

and by virtue of (8)

$$\frac{\partial(\log N)}{\partial x_0} < \frac{1}{\log \left(\frac{8R}{a\epsilon^2} \right)} \frac{\int\int \frac{(x_0 - x)dxdy}{(x_0 - x)^2 + (y_0 - y)^2}}{\int\int f(x, y)dxdy},$$

$$\frac{\partial(\log N)}{\partial y_0} < \frac{1}{\log \left(\frac{8R}{a\epsilon^2} \right)} \frac{\int\int \frac{(y_0 - y)dxdy}{(x_0 - x)^2 + (y_0 - y)^2}}{\int\int f(x, y)dxdy}.$$

Thus by making $\log (8R/a)$ large the maximum variation in $\log N$ can

be made negligible. This fact is not by any means self-evident because the magnetic field at an infinitesimal distance from a current is infinite.

Therefore, in coils with sufficiently large diameter the magnetic flux through any one turn is the same as that through any other.

These results will now be applied to the case of a short closely wound solenoid.

(*a*) *Short Closely Wound Solenoid.*—Let *A B C D* (Fig. 4) be a diametral section of a circular cylinder, the axis of the cylinder being *E F*. The coil is imagined to be wound on the surface of the cylinder, each turn being circular, the turns being wound close together and the number of turns being large. The distance *A B* is taken to be negligible in comparison with the distance *A C*.

In the diametral plane points will be named by means of a cartesian system of axes *OX*, *OY*. The origin *O* of the system is placed at the middle of the line *AB*. *OX* is along *A B*, and *OY* is perpendicular to *A B*. The number of turns being large the current is uniform within a few turns. Thus the x coördinate of a turn may be chosen as the parameter previously denoted by x. The width of the coil is denoted by $2a$.

It was shown that the flux through one turn of the coil is the same as that through any other. *Therefore in* (6) *the quantity* $M(x)$ *is a constant.* Since

$$\int_{-a}^{+a} M(x)dx = L,$$

$$\frac{M(x)}{L} = \frac{1}{2a}.$$

Hence

$$C_0 = \frac{1}{2a}\int_{-a}^{+a}\left\{\int_{-a}^{x}\frac{\alpha(x)}{L}dx\right\}dx. \qquad (10)$$

It now remains to compute the quantity $\alpha(x)$. This depends on the objects surrounding the coil. A simple condition is that in which all these objects are removed to an indefinite distance. However, practically this is not always realized. For this reason in the design of a standard inductance with a definite distributed capacity it seems advisable to surround the coil with a metallic shield. The mathematics in the special instance considered will be simplest if the shield is in the form of a solid of revolution obtained by constructing an ellipse whose foci are at *A* and *B* and whose plane is that of *A B C D*, and revolving the ellipse about the axis *E F*. The potential of the shield will be arbitrarily taken as zero. The middle of the coil (*i.e.*, the point *O*) is considered as connected to the shield by a short wire so that the potential of *O* is also zero.

Then by virtue of (3) the potential at x on $A\ B$ is

$$V = -\frac{di}{dt} L \frac{x}{2a}, \tag{11}$$

the positive direction of i being from negative to positive values of x.

In order to find $\alpha(x)$ it is necessary to find $Q(x)$. This is attained by finding the distribution of potential between $A\ B$ and the shield.

The problem is solved without difficulty by the use of elliptical coordinates defined by the transformation

$$x + jy = a \cosh (u + jv), \tag{12}$$

where

$$j = \sqrt{-1}$$

or its equivalent:

$$\begin{aligned} x &= a \cosh u \cos v, \\ y &= a \sinh u \sin v. \end{aligned} \tag{13}$$

It follows from (12) that the real or the imaginary part of any monogenic function of $u + jv$ is a solution of Laplace's equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0.$$

Again if u is kept constant then in virtue of (13)

$$\frac{x^2}{a^2 \cosh^2 u} + \frac{y^2}{a^2 \sinh^2 u} = 1,$$

so that the point (x, y) lies on an ellipse whose foci are $(\pm a, o)$. Let then u_0 be the value of u which is assigned to the ellipse of the shield. Then

$$V = -\frac{L}{2} \frac{di_1}{dt} \frac{(\epsilon^{-u} - \epsilon^{u-2u_0}) \cos v}{1 - \epsilon^{-2u_0}}, \tag{14}$$

satisfied Laplace's equation because $(\epsilon^{-u} - \epsilon^{u-2u_0}) \cos v$ is the real part of $\epsilon^{-(u+jv)} - \epsilon^{-2u_0}\epsilon^{u+jv}$. Also if $u = u_0$, then $V = 0$, and if $u = 0$ $V = -L(di_1/dt)(x/2a)$ so that (11) is satisfied. Finally the above expression for V, as well as its first derivatives are finite and continuous in the space between $u = 0$ and $u = u_0$ except when the segment $A\ B$ is crossed. Consequently this is the potential which exists in the space under the conditions of the problem.

The segment $A\ B$ is given, of course, by $u = 0$. If u is slightly greater than 0 the segment expands into an ellipse. The half of the ellipse corresponding to positive values of y is given by the values of v between 0 and π. The negative values of y are similarly given by values of v between π and 2π. Thus the line AB is the limiting state of such an ellipse for which $u = \delta$, δ being infinitesimal.

Differentiating (12) one obtains:

$$\frac{d(x+jy)}{d(u+jv)} = a \sinh (u+jv) = a[\sinh u \cos v + j \cosh u \sin v];$$

$$\therefore \left|\frac{d(x+jv)}{d(u+jv)}\right| = a \sqrt{\sinh^2 u \cos^2 v + \cosh^2 u \sin^2 v}. \qquad (15)$$

The surface density of charge on the ellipse $u = \delta$ is

$$-\frac{10^{-11}K}{35.956\pi}\left(\frac{\partial V}{\partial n}\right)_{u=\delta},$$

where $\partial/\partial n$ is the directional derivative as to the normal drawn out from $u \neq \delta$. But by (15)

$$\frac{\partial V}{\partial n} = \frac{1}{a\sqrt{\sinh^2 u \cos^2 v + \cosh^2 u \sin v}}\frac{\partial V}{\partial u}.$$

Therefore the surface density

$$\sigma_1 = -\frac{10^{-11}K\left(\frac{\partial V}{\partial n}\right)_{u=\delta}}{35.956\pi a\sqrt{\sinh^2 \delta \cos^2 v + \cosh^2 \delta \sin^2 v}}. \qquad (16)$$

Substituting $2\pi - v$ for v in (16) it is seen that the surface density at the reflection of the point given by v in $A\,B$ is also σ_1. The sum of the two surface densities is then

$$\sigma = 2\sigma_1 = -\frac{10^{-11}K}{17.978\pi a}\frac{\left(\frac{\partial V}{\partial n}\right)_{u=\delta}}{\sqrt{\sinh^2 \delta \cos^2 v + \cosh^2 \delta \sin^2 v}}.$$

In the limit when $\delta = 0$ it is found from (14) that

$$\sigma = -\frac{10^{-11}KLl \coth u_0 \cos v}{35.956\pi a\,|\sin v|}\frac{di_1}{dt}.$$

Let l be the length of one turn of the wire of the coil. Then the charge in the infinitesimal interval $(x, x+dx)$ is

$$-\frac{10^{-11}KLl}{35.956\pi a}\coth u_0 \frac{\cos v}{|\sin v|}\frac{di_1}{dt}dx.$$

Thus by the definition of $\alpha(x)$ given in (4)

$$\alpha(x) = -\frac{10^{-11}KLl}{35.956\pi a}\cosh u_0 \frac{\cos v}{|\sin v|}.$$

Substituting in (6) and remembering that A corresponds now to $x = -a$,

$$C_0 = \frac{10^{-11}K}{8.989}\int_{-a}^{+a}\frac{1}{2a}\left\{\int_{-a}^{x}\frac{-Ll}{(4\pi aL)}\coth u_0 \frac{\cos v}{|\sin v|}\,dx\right\}dx.$$

The integration is effected without difficulty by the substitution

$$x = a \cos v.$$

The result is

$$C_0 = \frac{10^{-11} Kl}{16 \times 8.989} \coth u_0 = 0.06952 \times 10^{-12} Kl \coth u_0. \qquad (17)$$

Since the major semi-axis of the ellipse $u = u_0$ is $a \cosh u_0$, and the minor is $a \sinh u_0$, the result could be also written as

$$C_0 = 0.06952 \times 10^{-12} Kl \frac{\alpha}{\beta} \text{ farads}$$

$$= 0.06952 Kl \frac{\alpha}{\beta} \mu\mu f,$$

where α = major semi-axis,

β = minor semi-axis.

In particular if the major and minor semi-axes are both made infinite the ratio α/β becomes unity. Thus the *effective capacity of a short single layer solenoid used with its middle grounded and undisturbed by surrounding objects is*

$$C_0 = 0.06952 Kl \mu\mu f = \frac{Kl}{16} \text{ cgs electrostatic units of capacity.} \qquad (19)$$

It is remarkable that according to the formula (18) the only modification introduced by the shield is given by the factor α/β which even for comparatively narrow shields becomes approximately 1.

(*b*) *Short, Closely Wound Solenoid Grounded at Terminal When in Shield.*—Consider now the case when the terminal B is connected to the shield. The method of solution is almost identical with that of the preceding case. The only difference is that the potential at any point, x, is

$$-\frac{L(x-a)}{2a}\frac{di_1}{dt}$$

rather than

$$-\frac{Lx}{2a}\frac{di_1}{dt}.$$

It is clear that if on the solution given by (14) a solution for Laplace's equation which is 0 at $u = u_0$ and $(L/2)(di/dt)$ at $u = 0$ be superposed, the result will satisfy all the conditions of the problem. The expression

$$\frac{L}{2}\frac{di}{dt}\left(\frac{u_0 - u}{u_0}\right)$$

actually vanishes when $u = u_0$ and becomes $(L/2)(di/dt)$ when $u = 0$.

Therefore,

$$V = \frac{L}{2}\left[\frac{u_0 - u}{u_0} - \frac{\epsilon^{-u} - \epsilon^{u-2u_0}}{1 - \epsilon^{-2u_0}}\cos v\right]\frac{di}{dt} \tag{20}$$

is the required solution for the potential.

From (20)

$$\left(\frac{\partial V}{\partial u}\right)_{u=0} = \frac{L}{2}\left[-\frac{1}{u_0} + \coth u_0 \cos v\right]\frac{di}{dt},$$

so that

$$\alpha(x) = -\frac{10^{-11}KLl}{35.956\pi a}\left[-\frac{1}{u_0} + \coth u_0 \cos v\right]\frac{1}{|\sin v|} \tag{20a}$$

and

$$C_0 = \frac{10^{-11}L}{17.978a}\int_{-a}^{+a}\left\{\int_{-a}^{+x}\frac{l}{4\pi a}\left[\frac{1}{u_0} - \coth u_0 \cos v\right]\frac{dx}{|\sin v|}\right\}dx$$

$$= 0.06952 \times 10^{-12}Kl\left[\coth u_0 + \frac{2}{u_0}\right]\text{farads}$$

or if it is preferrable to express the result in terms of α, and β, as before, then

$$C_0 = 0.06952 \times 10^{-12}Kl\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\frac{\alpha+\beta}{\alpha-\beta}}}\right]$$

$$= 0.06952Kl\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\frac{\alpha+\beta}{\alpha-\beta}}}\right]\mu\mu f. \tag{22}$$

In the derivation of this formula the reader probably noticed that the current at A was chosen as i_1 while the terminal B was grounded. The reason for this choice is that when the capacity of a condenser is measured one set of plates is connected to ground permanently and the other is connected to and disconnected from the measuring apparatus. Thus the charge measured (either directly or used indirectly in theoretical derivations of formulas) is the charge supplied to the *ungrounded plate*. This is, therefore, the charge Q_e used in the derivation of formula (6).

Care should be taken not to use the formula (22) in cases when $\alpha/\beta - 1$ becomes very small because then the dimensions of the ellipse becomes large and the equation

$$\frac{\partial^2 V}{\partial z^2} = 0$$

is no longer true.

Only if α is negligible in comparison with R is this legitimate.

For this reason it is essential to work out the distributed capacity of a short solenoid for the case when it is grounded at one terminal and is kept at a considerable distance from surrounding objects.

Short Single Layer Solenoid Grounded in Free Space.—Consider now the same coil removed from surrounding objects and grounded at some point by a fine wire. Let B be grounded. It is evident that

$$V_1 = \frac{-L}{2}\frac{di}{dt}\epsilon^{-u}\cos v \tag{23}$$

is an expression for the potential which gives correct differences of potential for the case considered. However, at B, $V_1 = 0$. In fact its value at B is

$$-\frac{L}{2}\frac{di}{dt}.$$

It is thus necessary to find a solution of Laplace's equation which becomes

$$+\frac{L}{2}\frac{di_1}{dt}$$

at the coil and which vanishes at infinity and then to add this solution to V_1. Or if it is hard to find this solution it is sufficient to know what distribution of charges on the coil will give rise to such a solution. The answer is given by the expression (7). This expression is interpreted most easily by writing

$$d' = \frac{d}{8R},$$

for then

$$V = -10^{-9}c^2\frac{2\rho_1}{K}\log d'.$$

This shows that if $8R$ were the unit of length V would be the same as is taken in the two-dimensional case when the wires are straight. Consequently, the distribution of charges is the same as in the two-dimensional case. Now in this case the charge density varies as $1/|\sin v|$, *i.e.*, as $1/\sqrt{a^2 - x^2}$, as is seen from (20*a*). Consequently such a distribution gives a uniform potential over $A\,B$. The exact value of this potential may be ascertained by letting the charge density

$$\sigma = \frac{\rho}{\sqrt{a^2 - x^2}}. \tag{24}$$

The potential of the coil due to this charge is obtained most easily at the center of the winding of the coil and as has been shown above is the same at the center as at any other point of the coil. It is

$$V_2 = \frac{8.989 \times 10^{11}}{K}\int_{x=-a}^{x=+a}\frac{2\rho\log\left(\frac{8R}{|x|}\right)}{\sqrt{a^2 - x^2}}dx = \frac{8.989 \times 10^{11}}{K}2\pi\rho\log\left(\frac{16R}{a}\right).$$

But this must be $+ (L/2)(di_1/dt)$. Therefore,

$$\rho = + \frac{10^{-11} KL \frac{di_1}{dt}}{35.956\pi \log\left(\frac{16R}{a}\right)}.$$

and the charge density corresponding to this value of ρ is

$$\cdots \sigma_2 = + \frac{10^{-11} K \frac{di_1}{dt} L}{35.956\pi \log\left(\frac{16R}{a}\right)\sqrt{a^2 - x^2}}$$

$$= + \frac{10^{-11} KL \frac{di_1}{dt}}{35.956\pi a \left\{\log\left(\frac{16R}{a}\right)\right\} |\sin v|}. \quad (24a)$$

The charge density called for by V_1 (see (23)) is

$$\sigma_1 = - \frac{10^{-11} KL}{35.956\pi a} \frac{di_1}{dt} \frac{\cos v}{|\sin v|}.$$

Hence the total charge density

$$\sigma = \sigma_1 + \sigma_2 = \frac{10^{-11} KL}{35.956\pi a} \frac{di_1}{dt} \left[\frac{1}{|\sin v| \log\left(\frac{16R}{a}\right)} - \frac{\cos v}{|\sin v|} \right]. \quad (25)$$

If as before l is the length of one turn of wire (*i.e.*, $l = 2\pi R$) then the charge in the infinitesimal interval $(x, x + dx)$ is

$$\sigma l x d = \frac{10^{-11} KLl}{35.956\pi a} \frac{di_1}{dt} \left[\frac{1}{|\sin v| \log\left(\frac{16R}{a}\right)} - \frac{\cos v}{|\sin v|} \right]$$

and

$$\alpha(x) = \frac{10^{-11} LlK}{35.956\pi a} \left[\frac{1}{|\sin v| \log\left(\frac{16R}{a}\right)} - \frac{\cos v}{|\sin v|} \right]. \quad (26)$$

Therefore (see (6))

$$C_0 = \frac{10^{-11} K}{35.956} \int_{-a}^{+a} \frac{1}{2a} \left\{ \int_{-a}^{x} \frac{Ll}{\pi a L} \left[\frac{1}{|\sin v| \log\left(\frac{16R}{a}\right)} - \frac{\cos v}{|\sin v|} \right] dx \right\} dx.$$

Performing the integrations:

$$C_0 = \frac{10^{-11}Kl}{8.989}\left[\frac{1}{16} + \frac{1}{8\log\left(\frac{16R}{a}\right)}\right] \text{ farads}$$
$$= 1.1124\left[\frac{1}{16} + \frac{1}{8\log\left(\frac{16R}{a}\right)}\right] Kl \text{ micromicrofarads.} \qquad (27)$$

The formulas (22) and (27) are both seen to be made up of two parts. The first part is the effective capacity of the coil when ungrounded. The second may, therefore, be called the effective capacity due to grounding.

In the following paragraph, it will be shown that *for the case of a coil used in free space the effective capacity due to grounding is $\frac{1}{4}$ of the electrostatic capacity of the coil to ground.*

Equation (24*a*) gives the surface density of charge which maintains the surface of the coil at a potential $\frac{L}{2}\frac{di_1}{dt}$. Therefore, the electrostatic capacity of the coil is

$$\int_{v=0}^{v=\pi} \frac{\sigma_2 la \sin v dv}{\left(\frac{L}{2}\right)\frac{di_1}{dt}} = \frac{10^{-11}K}{17.978\log\left(\frac{16R}{a}\right)}.$$

But in (27) the distributed capacity due to grounding is

$$\frac{1}{4}\,\frac{10^{-11}c^{-2}Kl}{17.978\log\left(\frac{16R}{a}\right)}.$$

Therefore, the distributed capacity due to grounding is one fourth of the electrostatic capacity to ground.

This must be true as long as the distribution of charge introduced by grounding is the same as in the case of a coil grounded in free space, *i.e.*, removed from surrounding objects and grounded by a fine wire. Since the only difference between (14) and (20) is given by the term

$$\frac{L}{2}\,\frac{u_0 - u}{u_0}\,\frac{di_1}{dt}.$$

The distribution of charge introduced by the grounding of a coil in an elliptical shield is of the form $A/|\sin v|$, where A is a constant. Comparing this with (24) it is seen that *the distributed capacity introduced by grounding of the coil in an elliptical shield is one fourth of the electrostatic capacity of the coil to the shield.*

Further, even if the coil is not closely wound, the distributed capacity due to grounding must still be approximately equal to one fourth of the electrostatic capacity to ground as long as the winding is sufficiently close to make the average distribution of charge the same as that of a coil with close winding.

Neglecting minor corrections it is easy to measure the electrostatic capacity of a coil to ground in given surroundings. For this purpose a circuit L, C, Fig. 5, is tuned to the frequency of a generating set A and the reading of the shielded condenser C is noted. The shield of C is grounded and a wire BD extends permanently to the coil L' whose electrostatic capacity to ground is to be measured. By a slight motion of the end D, contact is made with L' and C is retuned. The difference between the capacity of C at the two settings gives the capacity of L' to ground. L' is of course not connected to ground directly during the measurement.

This has been verified experimentally. Thus for a coil for which the distributed capacity when ungrounded is 30 micromicrofarads the capacity when grounded came out 42 micromicrofarads while the measured electrostatic capacity to ground is 47 micromicrofarads. And it is seen that

$$30\ \mu\mu f + \frac{47\ \mu\mu f}{4} = 42\ \mu\mu f \text{ (approximately).}$$

The formulas derived are generally in satisfactory agreement with experiment if the capacity of leads is taken into account and the effect of condenser shields is made negligible. It is also of interest to note that formula (27) gives values for C_0 which are of the order of magnitude of the radius. This agrees with data accumulated at the Bureau of Standards.

Explanation of Constancy of C_0.

So far we have not explained why the number C_0 in (1) comes out a constant. In this section it will be shown that in the limit when the width of a coil becomes negligible compared to the diameter C_0 becomes a constant.

It was seen that the magnetic flux through all turns of the coil becomes the same under these conditions. Let then (L/N^2) be a constant such that the magnetic flux through a turn due to a current i in another turn is

$$\frac{L}{N^2} i,$$

N being the total number of turns. Then the flux through any one turn when i flows in all turns is

$$N\frac{Li}{N^2} = \frac{Li}{N}$$

and, therefore, the total flux through the coil is

$$N\frac{Li}{N} = Li$$

when the current is uniform. Thus L is the inductance of the coil.

Since the flux is the same through all turns, even if the current is not uniform the E.M.F. induced in a turn is

$$\frac{-L\left(\frac{di}{dt}\right)}{N},$$

where $\bar{\imath}$ is the average value of i, *i.e.*,

$$\bar{\imath} = \frac{i_1 + i_2 + \cdots + i_N}{N}, \tag{28}$$

i_1, $i_2 \cdots i_n$ being the currents in the 1st, 2d, 3d $\cdots$ Nth turns respectively. Thus the charge on the 1st, 2d, $\cdots$ Nth turns is

$$\frac{-f_1 L}{N}\frac{d\bar{\imath}}{dt}, \qquad \frac{-f_2 L}{N}\frac{d\bar{\imath}}{dt} \cdots \frac{-f_N L}{N}\frac{di}{dt},$$

where $f_1, f_2 \cdots f_N$ are independent of the frequency.

Let i_0 be the current entering the first turn. Then

$$i_2 - i_0 = \frac{f_1 L}{N}\frac{d^2\bar{\imath}}{dt^2},$$

$$i_3 - i_1 = \frac{f_2 L}{N}\frac{d^2\bar{\imath}}{dt^2},$$

etc.

Consequently the excess of the current through any turn over i_0 is proportional to $d^2\bar{\imath}/dt^2$ and the current can be written as

$$i_n = -\varphi_n\frac{d^2 i}{dt^2} + i_0 = i_0 + \omega^2\varphi_n\bar{\imath}.$$

Here φ_n is independent of the frequency. According to (28)

$$\bar{\imath} = i_0 + \omega^2\bar{\imath}\bar{\varphi},$$

where

$$\bar{\varphi} = \frac{\varphi_1 + \varphi_2 + \cdots + \varphi_N}{N};$$

$$\therefore \bar{\imath} = \frac{i_0}{1 - \omega^2\bar{\varphi}}. \tag{28a}$$

The E.M.F. induced in the coil is then

$$\frac{-\frac{di_0}{dt}L}{(1 - \omega^2\bar{\varphi})}.$$

If Q is the charge on the condenser

$$\frac{dQ}{dt} = -i_0.$$

Hence

$$\frac{1}{LC} = \frac{\omega^2}{1 - \omega^2\bar{\varphi}}$$

or

$$\omega^2 = \frac{1}{L\left(C + \frac{\bar{\varphi}}{L}\right)}.$$

One can now write

$$C_0 = \frac{\bar{\varphi}}{L},$$

C_0 being independent of ω. Then

$$\omega^2 = \frac{1}{L(C + C_0)},$$

where C_0 is a constant, which proves that the relation (1) is exact as long as the flux is the same through all the turns.

It may be worth mentioning here that if a coil is represented as a line with distributed constants C_0 is not independent of ω.

It must be mentioned also that in any of the applications mentioned the effective capacity is unchanged if the cross section is turned through an angle with reference to the axis of the coil. Thus the formula for a short single layer solenoid applies also to a pancake coil of small depth.

CURRENT DISTRIBUTION.

It is possible to study the current distribution experimentally in coils by inserting non-inductive resistors at different points when the coil is tuned to resonance. It can be shown that if R_1, R_2 are the values obtained for the resistance by the resistance variation method[1] at two points (1) and (2) of the coil then

$$\frac{i_1}{i_2} = \sqrt{\frac{R_2}{R_1}},$$

where i_1, i_2 are the currents at (1) and (2).

By this method the curves of Figs. 6, 7, 8 were obtained. The curves of Figs. 7 and 8 show the dissymmetry introduced into the circuit by a galvanometer or by grounding. The coil experimented on in the case of the curves on Fig. 6 was a 4-foot coil wound with 14 turns of number 19 double cotton covered wire with 1 cm. spacing on a wooden frame. The frame was so designed that as little as possible of the wood was in the

field of the coil. The dissymmetry was avoided by using a very small galvanometer (Weston thermogalvanometer) and making the readings

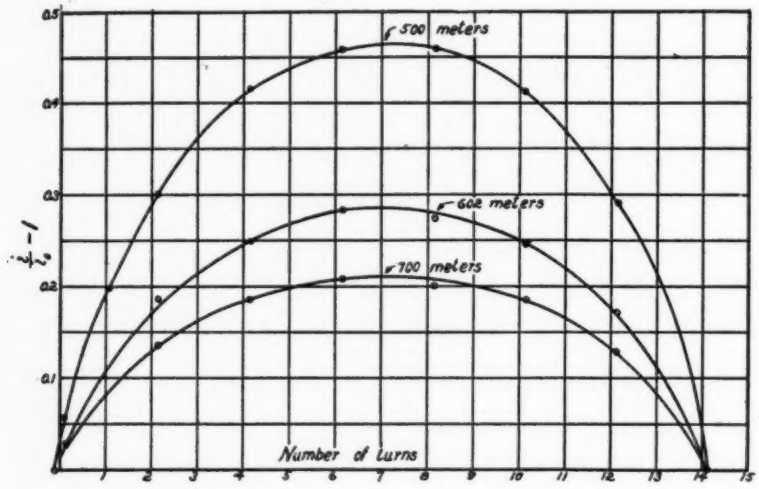


Fig. 6.

Current distribution in a coil determined experimentally at various wave-lengths.

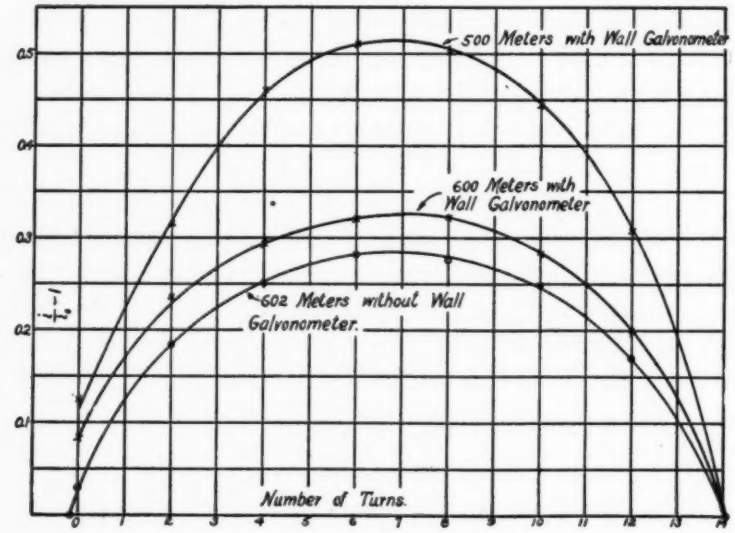

500 Meters with Wall Galvanometer
600 Meters with Wall Galvanometer
602 Meters without Wall Galvanometer.
Number of Turns.

Fig. 7.

Current distribution in a coil determined experimentally at various wave-lengths with slight dissymmetry caused by wall galvanometer.

from a large distance with a telescope. In the case of Fig. 8, i_0 was used for the current at the shielded condenser terminal.

[1] See Bureau of Standards Circular No. 74, p. 180.

It is interesting to see how this measurement (obtained during the course of some work at the Johns Hopkins University) verifies the theory. In the first place, since the width of the coil is small compared to the

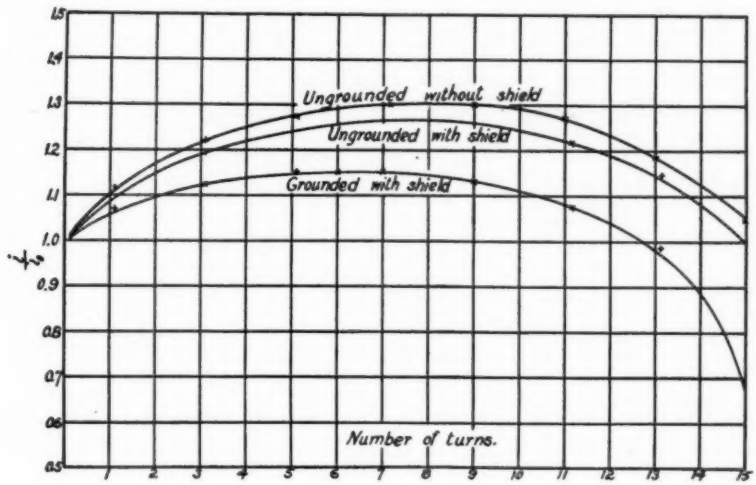


Fig. 8.
Effect of grounding and of condenser shield on current distribution.

diameter $(1/i_0) - 1$ must be nearly the same for all frequencies. Taking the experimental values for $(1/i_0) - 1$ at 700, 602, and 500 meters the following table is obtained

$\left(\frac{i}{i_0}-1\right)_{700}$	$\left(\frac{i}{i_0}-1\right)_{602}$	$\left(\frac{i}{i_0}-1\right)_{500}$	$\frac{\left(\frac{i}{i_0}-1\right)_{602}}{\left(\frac{i}{i_0}-1\right)_{700}}$	$\frac{\left(\frac{i}{i_0}-1\right)_{500}}{\left(\frac{i}{i_0}-1\right)_{700}}$
0.135	0.186	0.297	1.38	2.20
0.186	0.247	0.414	1.33	2.22
0.208	0.284	0.459	1.36	2.20
0.207	0.273	0.461	1.36	2.29
0.184	0.247	0.413	1.34	2.24
0.127	0.174	0.291	1.37	2.29
	Mean:		1.36	2.24

Measurements by the resistance-variation method are difficult to make with an accuracy higher than 1 per cent. This would give an accuracy of 0.5 per cent. in i/i_0 and worse than 10 per cent in $(i/i_0) - 1$. The largest deviation in the table is 2.3 per cent. in the case of the comparison between 602 and 700 meters; and 2.5 per cent. in the comparison between 500 and 700 meters. The agreement must thus be regarded as satisfactory.

Further a knowledge of the natural period of the coil enables one to calculate the ratios tabulated. In fact from (28*a*)

$$i - i_0 = \frac{i_0\bar{\varphi}\omega^2}{1 - \bar{\varphi}\omega^2} = \frac{i_0}{\frac{1}{\bar{\varphi}\omega^2} - 1}. \tag{29}$$

But

$$C_0 = \frac{\bar{\varphi}}{L}, \qquad i.e., \qquad \bar{\varphi} = LC_0.$$

Hence

$$\bar{\imath} - i_0 = \frac{i_0}{\frac{\omega_0^2}{\omega^2} - 1}, \tag{30}$$

where

$$\omega_0^2 = \frac{1}{LC_0}$$

and is the value of ω which would correspond to the natural frequency of a pure inductance L connected in series with a capacity C_0. In terms of wave-length λ

$$\bar{\imath} - i_0 = \frac{i_0}{\frac{\lambda^2}{\lambda_0^2} - 1}, \tag{31}$$

where

$$\lambda_0 = 2\pi\sqrt{LC_0}.$$

For the coil in question the value of l in (18) is 470 cm. Hence using the formula (19)

$$C_0 = 32.6\ \mu\mu f \text{ when } K = 1.$$

The measured value of C_0 with leads was 38 $\mu\mu f$ and the measured value of L_0 was 547 μh. The capacity of the leads was calculated as 3.4 $\mu\mu f$. Thus the measured value without leads is 34.6 $\mu\mu f$. The lead correction being uncertain 33.6 $\mu\mu f$ can be taken as the value of C_0. The corresponding value of λ_0 is 255 meters. Then for

	700 meters	602 meters	500 meters
$\frac{1}{\frac{\lambda^2}{\lambda_0^2} - 1}$ =	0.153	0.219	0.350

Here

$$\frac{0.350}{0.153} = 2.29,$$

$$\frac{0.219}{0.153} = 1.43.$$

According to the theory (Eq. 31) these must be the same as the measured quantities; viz.,

2.24 and 1.36.

It is seen that a difference exists between the measured and theoretical values. However, the difference is not large. It becomes less as the absolute value of the ratio increases and in the overall comparison between 500 and 700 meters it is 2.5 per cent. Although this is a serious discrepancy if we are concerned with current distribution, its effect on the constancy of the effective capacity of the coil is very small because 2.5 per cent. in 38 $\mu\mu f$ is barely 1 $\mu\mu f$ and is just within the limits of ordinary measurement.

SUMMARY.

1. The effective capacity of an inductance coil is defined as the constant C_0 in the equation

$$L(C + C_0) = \frac{I}{\omega^2},$$

which is known to be true experimentally.

2. A general formula is derived for the effective capacity. This is formula (6) of the paper.

$$C_0 = \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{x_1}^{x} \frac{\alpha(x)}{L} dx \right\} dx$$

Here x is an arbitrary parameter along the wire

$$\alpha(x) = \frac{Q(x)}{\left(\frac{di_1}{dt}\right)},$$

where $Q(x)dx$ is the charge between x and $x + dx$.
i_1 is the current at the terminal,
L is the self-inductance,
$M(x)dx$ is the mutual inductance of the element dx to the rest of the coil.

3. The formula is illustrated in special cases. These are: the short single-layer solenoid, used when grounded in elliptical shield, insulated in elliptical shield, insulated and grounded in free space.

The formulas for the cases discussed are:

(I.) Short single-layer solenoid one turn of which has a perimeter l or pancake of small depth used in elliptical shield of major semi-axis α and minor semi-axis β and insulated from shield

$$C_0 = 0.06952Kl\frac{\alpha}{\beta}\mu\mu f = \frac{Kl}{16}\frac{\alpha}{\beta}\text{C.G.S. electrostatic units.}$$

(II.) Same coil connected to shield at terminal

$$C_0 = 0.06952Kl\left[\frac{\alpha}{\beta}+\frac{2}{\log\sqrt{\frac{\alpha+\beta}{\alpha-\beta}}}\right]\mu\mu f$$

$$=\frac{Kl}{16}\left[\frac{\alpha}{\beta}+\frac{2}{\log\sqrt{\frac{\alpha+\beta}{\alpha-\beta}}}\right]\text{C.G.S. electrostatic units.}$$

(III.) Same coil used outside shield and insulated

$$C_0 = 0.06952Kl\mu\mu f = \frac{Kl}{16}\text{C.G.S. electrostatic units.}$$

(IV.) Same coil used outside shield and grounded

$$C_0 = 1.1124\left[\frac{1}{16}+\frac{1}{8\log\left(\frac{16R}{a}\right)}\right]Kl\mu\mu f$$

$$=\left[\frac{1}{16}+\frac{1}{8\log\frac{16R}{a}}\right]\text{C.G.S. electrostatic units.}$$

R being the radius. All the formulas with the exception of the last apply not only to coils having circular turns but also to other shapes such as square, rectangular, etc.

4. An explanation is given of the remarkable constancy of C_0 as found by experiment in the case of short coils.

5. An experimental verification of the theory is given by direct measurement of capacity and by investigating the current distribution in a coil at various frequencies.

The author is very grateful to Dr. F. W. Grover, Dr. C. Snow, Dr. J. H. Dellinger, Mr. L. E. Whittemore, Dr. J. S. Ames, Dr. H. L. Curtis, and Mr. C. N. Hickman for reading the manuscript.

APPENDIX.

The approximation used here is

$$F(k) = \log\frac{4}{\sqrt{1-k^2}}.$$

In order to see the truth of this expression use can be made of Maxwell's formula for the mutual inductance of two coaxial circles whose radii are both approximately a and whose shortest distance apart is b. This is in electromagnetic units in air.

$$M = 4\pi a\left[\log\left(\frac{8a}{b}\right)-2\right].$$

The above expression is only approximately true. The expression will now be equated to Maxwell's exact formula in elliptic integrals in its limiting form and hence the value of $F(k)$ for small $k - 1$ will be derived. This exact formula for two concentric circles of radii a_1, a_2 is in the same units.

$$M = 4\pi \left[\frac{a_1^2 + a_2^2}{a_1 + a_2} F\left(\frac{2\sqrt{a_1 a_2}}{a_1 + a_2}\right) - (a_1 + a_2) E\left(\frac{2\sqrt{a_1 a_2}}{a_1 + a_2}\right) \right].$$

If a_1 is nearly a_2 it becomes approximately

$$M = 4\pi a \left[F\left(\frac{2\sqrt{a_1 a_2}}{a_1 + a_2}\right) - 2E\left(\frac{2\sqrt{a_1 a_2}}{a_1 + a_2}\right) \right].$$

Also, since

$$\frac{2\sqrt{a_1 a_2}}{a_1 + a_2}$$

is nearly 1,

$$E\left(\frac{2\sqrt{a_1 a_2}}{a_1 + a_2}\right)$$

is nearly 1. Identifying the two expressions for M the approximation for $F(K)$ follows immediately.

A different proof is found in Whittaker and Watson, Modern Analysis, pp. 514–515, Sections 22–737.

Radio Laboratory,
Bureau of Standards,
Washington, D. C.

MAGNETIC SUSCEPTIBILITY OF NICKEL AND COBALT CHLORIDE SOLUTIONS.

BY LAURA BRANT.

SYNOPSIS.

The Magnetic Susceptibilities and Magneton Numbers of Nickel and Cobalt in Aqueous Solutions of their Chloride Salts.—The susceptibilities of solutions have been found by a modified form of the "Kelvin-Wills" method. Data is given for nickel chloride solutions ranging in molar concentration from 0.001000 to 3.765 and for cobalt chloride solutions from 0.001018 to 2.0353, for field ranges of 3,000 to 14,000 gausess for the lower concentrations and 3,000 to 10,000 for the higher. All measurements were made at a temperature of 20° C.

The susceptibilities of the salts have been computed from the susceptibilities of the solutions by application of the Wiedemann law, and the susceptibilities of the metals obtained by extending the Wiedemann relation to the salts.

The susceptibilities of the solutions measured were found to be independent of the field strength throughout the ranges of field used, and the susceptibilities of the salts, both nickel and cobalt, were found to be independent of the concentrations of the solutions. The value found for the molecular susceptibility of nickel is 0.004423 and of cobalt 0.010362. These values give upon application of Weiss's equation 16.0 and 24.5 magnetons respectively for the nickel and cobalt atoms.

Susceptibility of Air.—An auxiliary result obtained in the experiments is that the volume susceptibility of air at 20° C. and average conditions as regards pressure and moisture extending over a number of months is 0.0288×10^{-6}.

FROM a consideration of the values of Pascal[1] and of Liebknecht and Wills[2] for the magnetic susceptibility of paramagnetic salts in solution Professor Weiss conjectured[3] that the molecular magnetic moment was an integral multiple of an elementary moment which he called the "magneton." The existence of this natural magnetic unit is fundamental in his magneton theory. Whatever may have been, or may be, the fate of this theory, a result of it was a quickened interest in the magnetic study of solutions, a study which had held the attention of experimenters since the classical magneto-chemical investigations of Wiedemann. Of the large literature which has followed, five publications are of special interest in connection with the subject of this paper. Two of these deal with the susceptibility and magneton number of nickel in its salts in aqueous solutions and three with the susceptibility and magneton number of cobalt in aqueous solutions of its salts.

[1] Ann. Chim. Phys., 8e Ser., T. xvi, p. 531; 1909: T. xix, p. 5; 1910.

[2] Ann. d. Phys., Bd. I., p. 178; 1900.

[3] Jour. de Phys., 5e Ser. T. 1, p. 965, 1911.

The first of these papers is by the Spanish physicists Cabrera, Moles, and Guzman,[1] and gives the results of precision work upon the normal salts, nickel sulphate, nickel chloride, and nickel nitrate. Measurements were made upon solutions varying in concentration over a considerable range. In the case of nickel chloride this was from 0.02208 to 0.34630 grams of salt per gram of solution. The second paper gives the values obtained from the careful measurements of Weiss and Mlle. Bruins[2] upon solutions of the same normal salts. The concentrations used for the case of nickel chloride ranged from 22.690 to 0.623 per cent. of nickel chloride. The results for the susceptibility of the solutions in both of these researches were obtained by means of a modified form of Quincke's ascension method, after the manner of Piccard in his determination of the susceptibility of water; and the value for the metal content found by an application of Wiedemann's law. The conclusions from the two experiments were in good accord. Both led to a susceptibility coefficient for nickel which was independent of the concentration, and to an integral number, 16, of magnetons for the nickel atom.

Of the papers on cobalt, the first gives the results of measurements by Cabrera, and his co-workers, Jimeno, and Marquina[3] on solutions of cobalt chloride, cobalt sulphate, and cobalt nitrate, obtained, as in the case of the nickel solutions, by Piccard's modification of Quincke's classical method, and application of the Wiedemann law. The susceptibility of cobalt was found to vary with the concentration. Fractional numbers of magnetons between 24 and 25 resulted, but in such a manner that whole numbers were approached as limiting values. Thus there were found 24 magnetons for the cobalt atom from measurements on the more dilute solutions, and from measurements on the stronger concentrations, 25. The second paper dealing with cobalt is by Trümlper,[4] a student at Zürich, who had completed two independent series of measurements on solutions of the chloride, nitrate, and sulphate salts at the time the earlier publication appeared. The first of these series was made by means of a magnetic balance of the Curie-Cheneveau type and resulted in a magnetic coefficient for cobalt which varied with the concentration. Comparison later with the work of Cabrera, Jimeno, and Marquina showed that there was not a close quantitative agreement, but that Trümpler's values were, nevertheless, consistent with the interpretation given by the Spanish physicists and pointed to the same integral numbers, 24 and 25, of magnetons for the cobalt atom in limiting conditions in

[1] Sci. Arch., p. 325, 1914.

[2] Amst. Akad. Proc., p. 246, Vol. 18.

[3, 4] See "Le magneton et les sels paramagnetiques dissous," by P. Weiss, Revue Gen. de L'El., p. 925, Dec. 15, 1917. These papers are known to the writer only through this review.

solution. For the second series of measurements Trümpler also adopted the Quincke-Piccard ascension method. These measurements gave a value of the susceptibility of cobalt which did not depend upon the concentration of the solution, and a fractional number, approximately 24.5, of magnetons for the cobalt atom. The third paper contains results for cobalt obtained by Quartaroli[1] from dilute solutions of the same three cobalt salts by means of his differential method, a nickel chloride solution having been used as standard. Under the condition that the magneton number of nickel is a constant, 16, the results for the chloride solutions gave for cobalt a molecular coefficient which increased when the concentration was increased, and a magneton number ranging from about 24.5 to 25.3 for the concentrations used.

Experiments for determining the susceptibilities and magneton numbers of nickel and cobalt in aqueous solutions of their chloride salts by a method which differs from those mentioned above had been practically completed in this laboratory before Weiss's review (l.c.) came to the writer's attention. The experiments were entirely finished before Quartaroli's paper was known. To present the results obtained by the method here employed is the object of the present paper.[2]

METHOD.

Lord Kelvin[3] suggested, as a means of determining the susceptibility of diamagnetic and feebly magnetic solid substances, placing a suitably shaped portion of the substance, such, *e.g.*, as a cylinder or a rectangular slab, having known dimensions in a non-uniform magnetic field and measuring the mechanical force exerted by the field upon it. He observed that the field strength, which must be known, could be obtained conveniently through finding the force acting upon a chosen length of wire carrying a measured current across the magnetic lines of force. The susceptibility could then be computed from known relations. Professor Wills[4] determined the susceptibility of solids by a method in which these suggestions are embodied and pointed out that the process was applicable to measurements on liquids and gases. He later used it, adapted to liquids, in determining the susceptibility of water, and others have since made use of it in the study of other liquids. A particularly favorable feature of the method as developed by Professor Wills is that the determination of the field is made at the same time that the force upon the body is found.

[1] Soc. Chim. Ital, Gazz., 48[1], p. 79, 1918.

[2] The measurements reported in this paper were completed during the summer of 1919.

[3] Brit. Assoc. Report, p. 745, 1890.

[4] PHYS. REV., *6*, p. 223, 1898.

The method of obtaining the susceptibility of the solutions in the present experiments is also based upon the suggestions of Lord Kelvin. The method follows a different procedure for obtaining mechanical force than was used in the earlier adaptation by Professor Wills, but borrows from that adaptation the details for the field determination. In the earlier application the mechanical force was found by means of a balance from one arm of which a rectangular slab of the body, and the current element, were suspended in the field: in these experiments the force has been determined from the currents required to carry the body to positions of known displacements when suspended as a pendulum in the field.

Wiedemann's law has been applied in computing the susceptibility of the metal from that of the solution, and Weiss's equation in computing the number of magnetons.

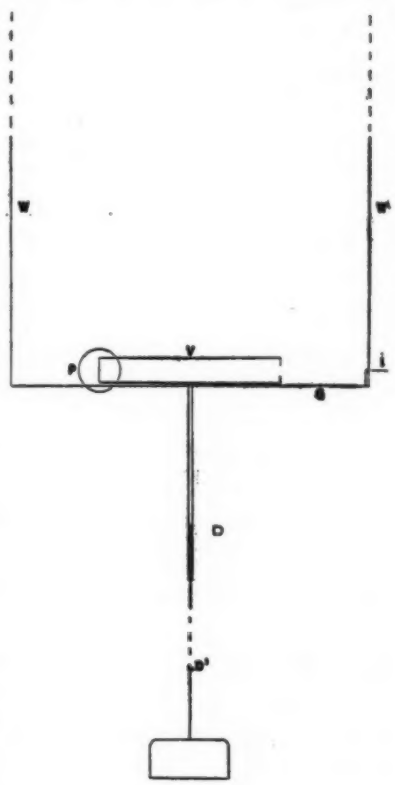


Fig. 1.

Apparatus.

The line drawing (Fig. 1) gives a general idea of the distribution and relative size of the parts of the apparatus making up the suspended system. A narrow glass platform, *G*, upon which a rectangular glass vessel, *V*, for containing the liquid, rested by means of a point, dot, slot device (not indicated in the figure), was suspended horizontally between the poles, *P*, of an electromagnet by four fine copper wires *w*, *w*, *w'*, *w'*, of equal length which sustained equally the weight of the platform and its load. The wires *w* and *w'* passed through the capillary bore of small glass tubes (2.2 cm. in length) attached to the platform, and through two similar but longer tubes (9.7 cm. long) attached to a brass bar supported above. The tubes were parallel and symmetrically placed at the ends of the platform and bar, respectively, at right angles to their length. The ends of the tubes, forming the points of suspension, were well rounded to reduce friction, and slipping of the wires was prevented by a touch of china cement in the capillary bore. The planes *w*, *w* and *w'*, *w'* were parallel and 20.4 cm.

distant, and were cut by the horizontal planes containing the upper and lower suspension points to a length of 81.50 cm. The position with respect to the poles was such as to bring the length of the vessel, which was parallel to the long axis of the platform, perpendicular to the lines of force and one end (in the absence of a magnetic force), approximately in the position of the strongest part of the field. The other end was well out of the field.

The cell, V, measured about 10.4 × 1.4 × 1.0 cm. externally. Around the top, the end to be placed in the field, and the bottom was pasted a strip of tinfoil which served to conduct a current across the lines of force, the length of the foil across the field end of the cell forming an element of circuit used in the field determination. The ends of the tinfoil were in contact, at the outer end of the cell, with short lengths of thin copper sheet to which were soldered a few cm. of brass wire. Small slotted brass tips slipped over the ends of the wires connected the ends of the tinfoil to fine silk-covered strands of a long spiral of two-ply copper twist, which hung vertically from the brass support above, and formed part of a current circuit described further in a following paragraph. The width of the foil was equal to the internal width of the vessel. The field end of the vessel was cut from very thin glass, so that the tinfoil was separated from the inner surface by a small fraction of a mm. only. The side walls were about 1 mm. in thickness, and the outer end, which was bored to permit of filling and emptying the vessel, was about 3 mm. thick. When filled with liquid the cell was sealed by means of a thin coating of paraffin.

D and D' (of brass and copper respectively) are parts of an electromagnetic damper which was in position but not used when the readings recorded in this paper were taken.

A glass upright, at the outer end of the glass platform, supported a vertically ruled glass index, i, the plane of the rulings being parallel to the length of the platform. In the early part of the work the index was a square of glass lined with very light diamond traces about 0.5 mm. apart. Later this was replaced by the scale of a Bausch and Lomb stage micrometer ruled to 0.01001 mm.

The brass bar which supported the whole pendular system was pinned in a manner permitting rotation about a central axis perpendicular to its length to a brass plate the elevation and tilt of which was adjustable by means of screws; the plate was mounted upon two crossed, screw-controlled, guided brass slides, providing adjustment parallel and perpendicular to the field. A thermometer graduated at 0.2° C. hung from the guide above so that its bulb was just above the cell. The whole

was borne by a brass arm extending from a heavy iron casting which was firmly bolted to the base of the magnet. The entire system was enclosed in a well-lighted wooden box as a protection against draughts.

A second iron casting bolted to the base of the magnet and finished with brass fittings furnished a rigid support and the necessary means of adjustment for keeping a microscope at the level of the index, *i*, and in focus upon the scale. A stout brass rod bracing the two castings against each other effectively eliminated any disturbing motion of either due to the field. The microscope was provided with a filar micrometer having a screw head divided into one hundred parts. For the optical combination as adjusted throughout the experiment, one division of the head of the screw was equivalent to 0.0000588 cm.; one tenth of a division could easily be estimated. A Leitz Wetzlar object micrometer reading to 0.01 mm. was used for the calibration. The observations of the index were made by means of the microscope through a window of very thin clear mica in the box enclosing the suspension.

The electromagnet used was of the Du Bois half-ring type. The core was of about 10 cm. diameter and was fitted with right conical pole pieces having a base angle of approximately 30° and faces initially 1.5 cm. in diameter. These faces were later cut away giving new surfaces about 2.5 cm. across. The distance between the faces was about 1.3 cm. The coils had a capacity of 25 amperes. Current was supplied by storage batteries at about 110 volts. A reversing key provided a means of demagnetizing the magnet. The magnet was mounted on foot-screws which rested upon steel rails placed across the top of a pier built up from the cement floor of a sub-basement chamber.

For some time before measurements were to be made, the temperature of this chamber was kept at approximately 20° C. by means of an adjustable electric heater which supplemented the steam heat provided in the building; due to the subterranean location of the room the temperature seldom went above 20° C. even in the hottest days of summer.

The upper end of the copper spiral previously mentioned was led out of the wooden box at the top and was then connected through a commutator, in series with storage batteries and resistances adapted to sending a desired current through the foil. A Weston milliammeter provided with shunts and a Siemens-Halske milliammeter placed in the circuit were used in determining the strength of the current, the instrument read for any particular value being the one giving the most favorable range of scale. Both ammeters were carefully calibrated by means of a potentiometer and the two were checked against each other at frequent intervals.

SOLUTIONS.

The nickel and cobalt salts used were pure products obtained from Kahlbaum, and were found, upon analysis in the Chemical Laboratory of this University, to be free from cobalt and nickel respectively and from other ferromagnetic material. Conductivity water was used for the solutions. The nickel chloride was analyzed by Professor Fales of the department of chemistry at Columbia. The solutions of this salt were made up and analyzed by Professor Fales about two years before being used and had been carefully kept in non-soluble glass bottles. The densities were determined shortly before the susceptibility measurements were made. The cobalt salts were analyzed and new solutions made up for this work under the direction of Professor Beans, also of the department of chemistry at Columbia. In preparing the solutions a stock solution of the highest concentration was first made up. The lower concentrations were prepared from this by dilution. The concentrations of the several solutions were later determined by analysis. These determinations were for a temperature of $20 \pm .005°$ C. The data giving the concentrations were supplied by the chemists. The water used in the magnetic measurements was from the same conductivity water (this having been preserved together with the solutions), from which the nickel solutions had been prepared.

The densities were obtained by means of a pycnometer of about 10 c.c. capacity. They were found for a temperature well within $20 \pm .02°$ C.

THEORY OF METHOD.

The theory of the method may be expressed in terms of well-known relations. Any magnetic substance placed in a non-uniform magnetic field is acted upon by a mechanical force due to the field. Considering Maxwellian stresses in the media, it can be shown that the resultant force, F_l, acting upon a liquid slab contained in the vessel, V, is, for the position of the vessel given in these experiments, a force in the direction of the length of the vessel and of magnitude

$$F_l = \tfrac{1}{2}KA(H_1^2 - H_0^2), \tag{1}$$

where A is the area of cross-section of the slab at right angles to the direction of F_l, K the product of the permeability of the medium in which the system is suspended and the susceptibility of the liquid against the medium, and H_1 and H_0, with close approximation, the impressed magnetic forces at the ends of the slab in the stronger and weaker parts of the field respectively.

On account of the symmetry of the suspended system with reference

to the central vertical plane parallel to the pole faces, the resultant force, F, due to the field effective in deflecting the system has also the direction of the length of the vessel. If F_t and F_c denote this force when the vessel is filled with liquid and when empty, respectively, the difference between these forces for a given field is a measure of the force due to the field upon the liquid:

(2) $$F_l = F_t - F_c$$

for a given field.

Let f be defined as any force acting upon the system in the direction of the length of the vessel. (F_t and F_c are then particular values of f.) If d is the displacement of the system due to the force f in the direction of the force, elementary mechanics gives the following relation applicable for the case of small displacements and negligible stiffness of the suspension:

(3) $$f = \frac{M'gd}{l},$$

in which M' is the mass suspended, l the length of the suspension, and g the acceleration due to gravity.

If a current I amperes is sent through the tinfoil of length l' across the end of the vessel when the foil is placed in a field H_I there results, in accordance with electromagnetic theory, a force f given by

(4) $$f_I = \frac{H_I l' I}{10}.$$

With a given excitation of field suppose a current I', and again a current I'', to flow through the tinfoil. Let δ' and δ'' denote respectively the resulting displacements of the system. The displacements being due to the concurrent action of the field upon the system and upon the respective currents, we may equate the sum of the magnetic and electro-magnetic forces to the mechanical force, and thus obtain from (3) and (4)

$$F \pm \frac{H'l'I'}{10} = \frac{M'g\delta'}{l}$$

and

$$F \pm \frac{H''l'I''}{10} = \frac{M'g\delta''}{l},$$

where H' and H'' are the fields at the positions of the tinfoil at distances δ' and δ'' respectively from the undeflected position.

If the currents I are so directed that F and the respective forces f_I are opposing forces only the negative signs of the last two equations need be retained. If δ' and δ'' are small it may be assumed that $H' = H'' = H$,

where H is the field at the undeflected position of the tinfoil. Under these conditions the equations give

$$H = \frac{10M'g}{ll'} \frac{(\delta'' - \delta')}{(I' - I'')}$$

and

$$F = \frac{M'g}{l} \frac{(\delta''I' - \delta'I'')}{(I' - I'')}.$$

This method of determining H and F may be made all but a zero method since δ may be made small at will by suitably adjusting I.

When (as here) displacements are measured by means of a micrometer screw, it is convenient to write $\delta' = kn'$ and $\delta'' = kn''$ in which k is the value in cm. of one head division of the screw, and n' and n'' readings in head divisions. We have, then,

$$H = \frac{10M'gk}{ll'} \frac{(n'' - n')}{(I' - I'')} \tag{5}$$

and

$$F = \frac{M'gk}{l} \frac{(n''I' - n'I'')}{(I' - I'')}. \tag{6}$$

It is assumed (a) that the equalities $H' = H'' = H$ given above hold for the range of displacements used in these experiments. Assuming (b) that the field H_1 at the field end of the liquid slab is sensibly equal to the field at the position of the tinfoil, and (c) that H_0^2 is negligible in comparison with H_1^2, equations (1) and (2) give for the undeflected position of the system,

$$K = \frac{2(F_t - F_c)}{AH^2}. \tag{7}$$

Equations (5), (6), and (7) serve for computing K by the method of these experiments. C.G.S. units are used throughout.

If the atmosphere in which the system is suspended has a volume susceptibility κ_a, the volume susceptibility of the liquid, κ_l, is

$$\kappa_l = K + \kappa_a. \tag{8}$$

The specific (mass) susceptibility of the liquid, χ_l, is determined from this by the defining equation for χ_l, namely,

$$\chi_l = \frac{K_l + \kappa_a}{\rho_l}, \tag{9}$$

ρ_l being the density of the liquid.

Under the assumption that the additive relation expressed in Wiedemann's law[1] holds for the solutions used in these experiments, (an

[1] Ann. der Phys. und Chem., 126, 1, 1865.

assumption made tentatively but allowed to stand in view of the constancy of the respective results obtained for χ_s),[1] we have

$$m_1'\chi_1 = m_s'\chi_s + m_w'x_w, \tag{10}$$

in which m' denotes mass (in grams), χ specific susceptibility, and the subscripts l, s, and w refer to the solution, salt, and water respectively. If m is molecular weight, M the concentration (molar) of the solution, and L the number of c.c. in a liter, equation (10) may be written

$$\chi_l = \frac{m_s M}{L\rho_l}\chi_s + \left(1 - \frac{m_s M}{L\rho_l}\right)\chi_w. \tag{11}$$

Equations (9) and (11) give

$$\frac{m_s M}{L}\chi_s + \left(\rho_l - \frac{m_s M}{L}\right)\chi_w - \kappa_a - K_l = 0, \tag{12}$$

from which χ_s may be computed if both χ_w and κ_a are known, or if K_w is measured, ρ_w being given, and either χ_w or κ_a is known.[2]

In these experiments K_w has been determined, ρ_w taken from tables, and Piccard's[3] value for χ_w found for a temperature of 20° C. used.

Postulating that susceptibility is an additive (atomic) property of the dissolved salts[4] for the solutions used, we have in conformity with the Wiedemann law,

$$m_s\chi_s = m_m\chi_m + m_j\chi_j, \tag{13}$$

subscripts m and j relating to the metal and acid radical of the salt respectively. From this equation the molecular susceptibility, $m_m\chi_m$, may be computed if $m_j\chi_j$ is known. Pascal's[5] value for $m_j\chi_j$ (corrected) has been used.

Weiss's equation from which the number of magnetons has been found is:

$$N = \frac{\sqrt{3RT\chi_m}}{1123.5}, \tag{14}$$

[1] Wiedemann, himself, pointed out, and others have since noted again and again, that results for the susceptibility of the solute obtained by the application of this law were dependent upon the chemical state of the solution. Among the more recent papers of interest with reference to this (and to the additive relation, equation 13), are:

A. E. Oxley, Camb. Phil. Soc. Proc., Vol. 16, p. 421, 1912; Vol. 17, p. 65, 1912.

A. Quartaroli, Soc. Chim. Ital. Gazz., 46[1], p. 371, 1916, and the review by P. Weiss (l.c.).

[2] The three equations of the form (12) corresponding to three solutions of the same salt having different concentrations (for one of which M may be zero) may be solved, if χ_s and χ_w are constants, for χ_s and also χ_w and κ_a in terms of accepted constants, m, L, (and ρ_w) and the quantities measured in these experiments: the data given in this paper have not been found with the precision desirable for use in the solution of these equations.

[3] C. R., 155, p. 1234, 1912.

[4] See note 2 above.

[5] C. R., 148, p. 413, 1909.

in which R is the gas constant, T the absolute temperature, and 1123.5 the moment of the gram magneton.

MEASUREMENTS.

Test experiments showed the assumptions (a), (b), and (c) of the preceding section to be permissible. The values of M', l, l', and A were found by usual methods. The quantities n and I were obtained as follows: with a given mass suspended, preliminary trial gave the approximate value of I required for a chosen current through the magnet in order that the resulting electromagnetic force, f_I, and the force, F, due to the field should balance. This current I was then passed through the tinfoil and the field excited; the current through the foil was then so adjusted as to bring the index alternately to within n' and n'' units of the undeflected position, and the corresponding readings of the ammeter I_r' and I_r'' recorded. The displacements n' and n'' were observed through the microscope and recorded in terms of "head divisions" of the micrometer screw. The readings I_r' and I_r'' multiplied by the ammeter constant, s, gave the currents I' and I'' respectively. An odd number of settings (sometimes three, sometimes more) were made so that the initial and final readings were for the same position. A set of five or more readings could easily be obtained in a few seconds; the averages of the values for I_r' and I_r'' were used. These readings were taken at a temperature which did not differ from 20° C., as shown by the thermometer above the cell, V, by more than $\pm$ 0.2° C.[1] The determination of l, l', and A, and of M', n', n'', I_r' and I_r'' for the system with the empty cell in position and with the cell filled with each of the several liquids, together with the microscope and ammeter constants, k and s, completed the measurements required (the densities and chemical data being obtained) for the determination of the values $m_m\chi_m$ and N.

DATA AND COMPUTED VALUES.

The following have been taken as constants in this work:

Accepted Values.	*Measured Quantities.*
g = 980.2 cms/sec^2,	l = 81.50 cms.,
L = 1000 c.c.,	l' = 1.387 cms.,
R = 83.155 $\times$ 10^6 ergs/degree,	A = 0.848 cm^2.,

[1] A variation within a half degree or less in temperature had no noticeable effect upon the observations; but a temperature range of a few degrees resulted in marked differences in the values obtained. Since it was the aim in these experiments to determine the susceptibility at a constant temperature no quantitative work was done with the purpose of determining accurately the value of the temperature coefficient. Qualitative work however showed clearly that the coefficient was far greater for the glass or other parts of the suspended system han for the liquids used.

$\rho_w = 0.99823$ gms/c.c., $k = 0.00005883$ cm.
$\chi_w = -0.71927 \times 10^{-6}$,
$m_i\chi_i = -40 \times 10^{-6}$,

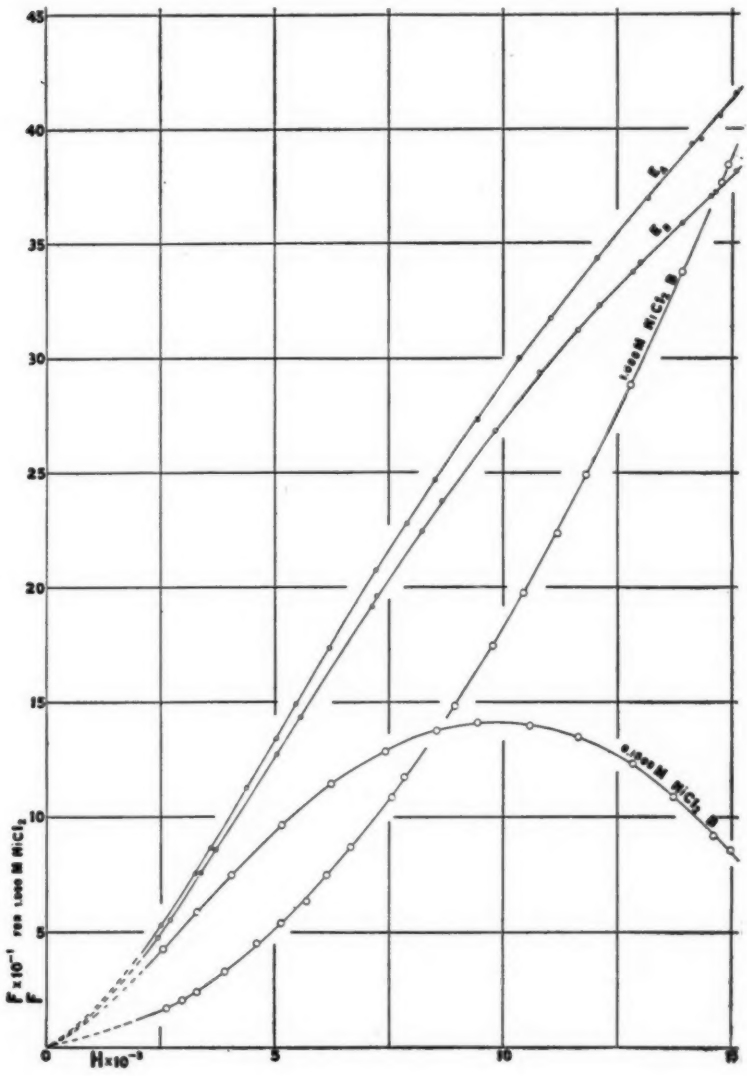


Fig. 2.

Figures 2–6 present graphically values obtained for F and H. Curves E are $F_c - H$ curves, others $F_t - H$ curves for the several liquids as indicated. Subscripts A, B, C, and D designate the particular $F_c - H$ curve, E, which obtained when the measurements giving the respective $F_t - H$ curves A, B, C, and D were taken. (In all cases points on the

$F_c - H$ curve were determined both before and after the observations for an $F_t - H$ curve were made.) The difference in the curves E are attributed to incidental changes in the apparatus.

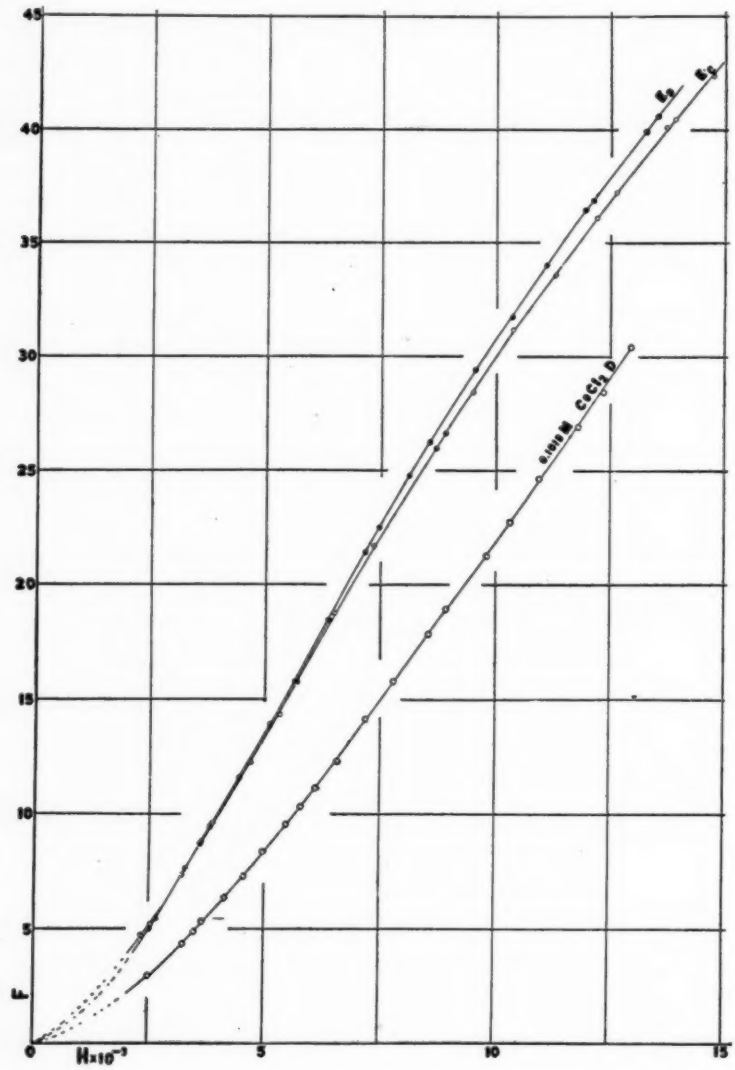


Fig. 3.

Tables I., II., and III. (curves E_A, H_2O A, and 0.001000 M $NiCl_2$ A) are illustrative of the 17 similar tables containing data from which, together with given constants, the $F - H$ curves were obtained.

Tables IV. and V. are representative of tables giving K for each of the liquids (including two for water). The values of H (at 1,000 gauss

intervals within the range used) and of F used in computing K are averages of several determinations read from curves so plotted that the fourth significant figures (where given) of the F columns were estimated tenths of millimeters along the axis of ordinates, and that estimated tenths of millimeters along the axis of abscissas corresponded to 5 units in the H columns. The several tables are summarized in Table VI.

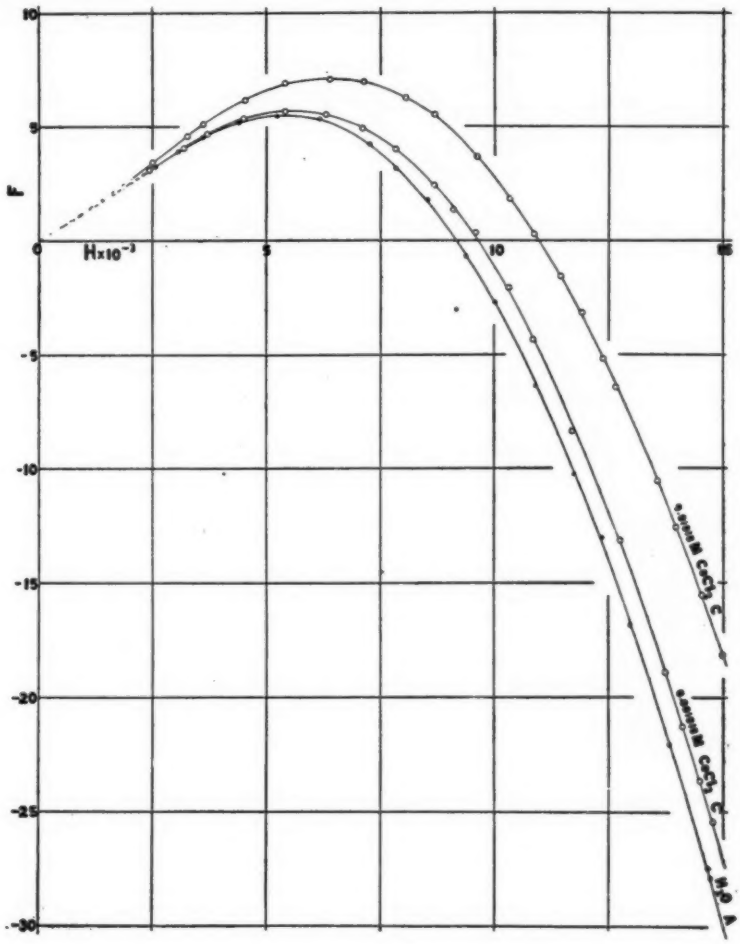


Fig. 4.

The densities ρ_l recorded in Tables VII. and VIII. are average values written, as in the case of the values of K, (Table VI.), with two deviation figures. Columns 5 and 6 of Tables VII. and VIII. give the values found by these experiments for the molecular susceptibility and magneton number of nickel and cobalt respectively. The deviations in the respective values are shown in columns 7.

If K_l, K_w, ρ_l, χ_w, and M are known to within approximately the same percentage of error, the effect upon $m_s\chi_s$ of the errors in χ_w and M are negligible in comparison with the errors in K_l, K_w and ρ_l (which contribute about equally to errors in $m_s\chi_s$) for the dilute solutions: for the

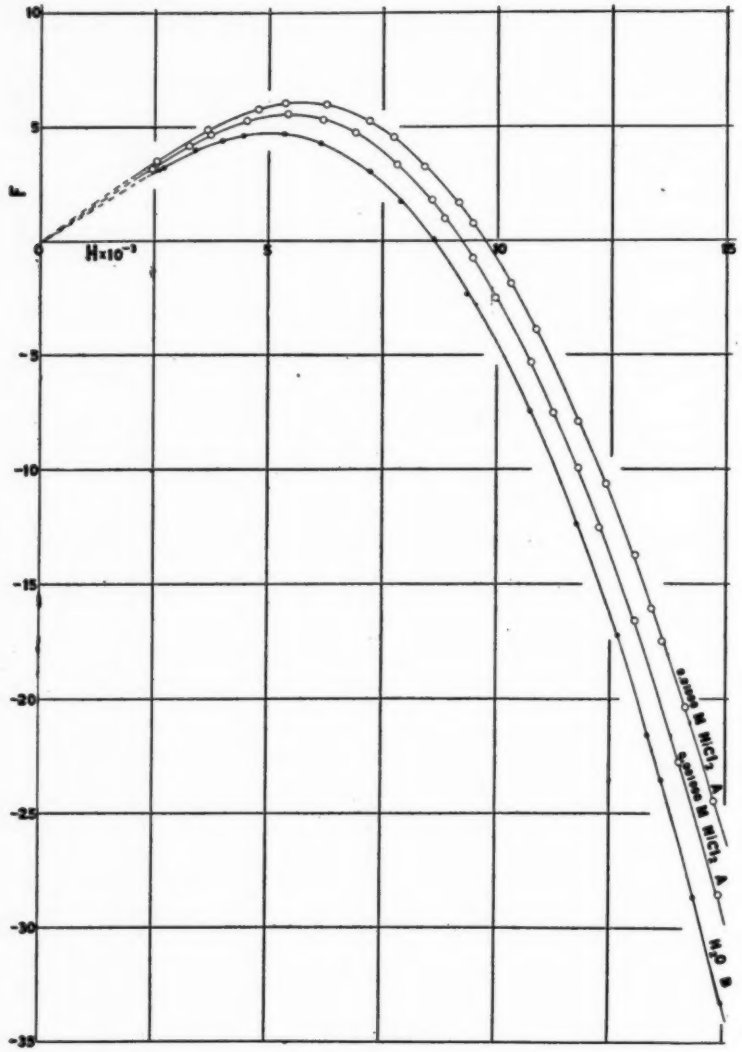


Fig. 5.

more concentrated solutions errors in K_w, ρ_l and χ_w are negligible in comparison with errors in K_l and M.

For the results in this paper, if the deviations in the values of K and ρ are taken as indicating the magnitude of errors in K and ρ, the value of

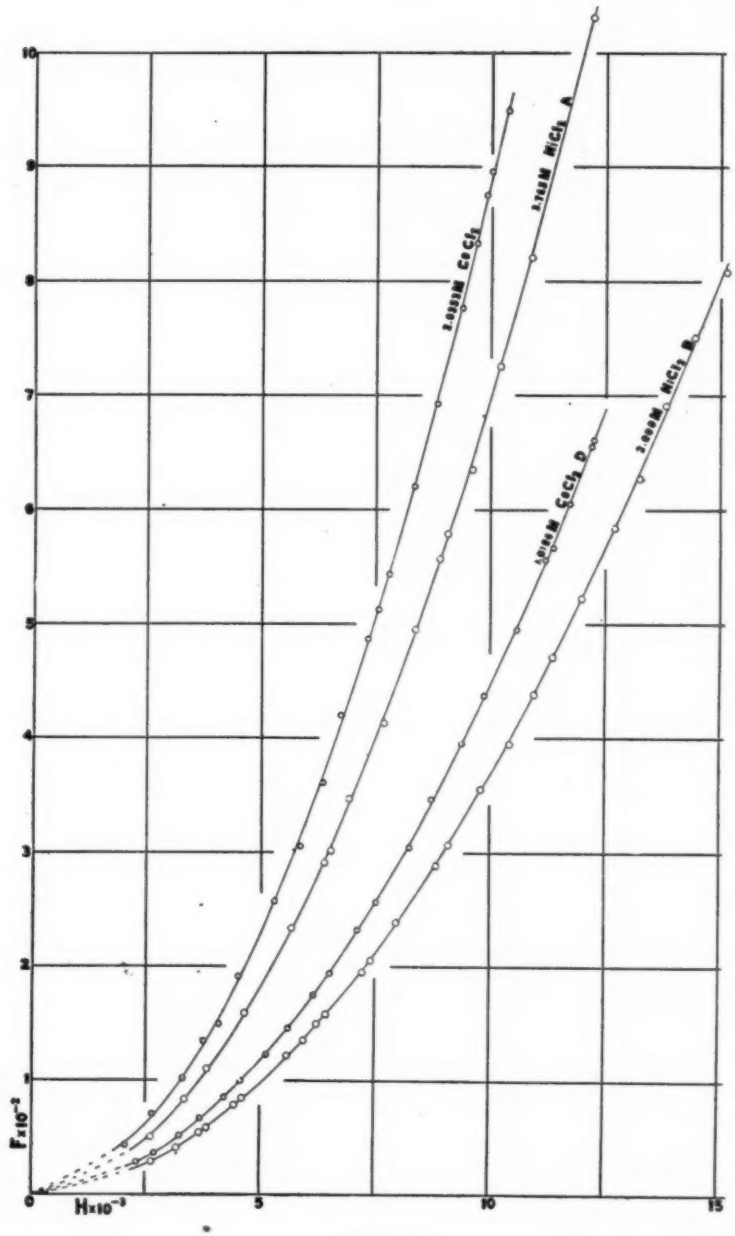

2.0000M CoCl2
2.0000M NiCl2 A
2.0000M CoCl2 B
2.0000M NiCl2 B
F×10⁻²
H×10⁻³
0
5
10
15

Fig. 6.

TABLE I.

Data for System with Empty Cell. Curve (E_A).

$M' = 51.412$ gms.

$n' = 0$; $n'' = 100$ head div.

$s = 2.674 \times 10^{-4}$. $t = 20°$ C.

I_v' (Amp/s).	I_v'' (Amp/s).	F_e (Dynes).	H (Gauss).
56.70	96.00	5.248	2,512
62.78	93.10	7.532	3,256
65.15	92.60	8.634	3,596
69.85	92.40	11.27	4,378
72.55	92.20	13.43	5,024
74.00	92.05	14.91	5,469
76.05	92.00	17.35	6,191
78.00	91.70	20.71	7,206
78.30	90.80	22.79	7,898
78.70	90.30	24.68	8,510
78.45	88.90	27.31	9,447
78.75	88.30	30.00	10,340
78.05	87.00	31.71	11,030
77.40	85.60	34.34	12,040
76.20	83.70	36.96	13,160
75.70	82.70	39.34	14,100
75.00	81.90	39.54	14,310
74.70	81.40	40.56	14,730
74.75	81.30	41.51	15,070

TABLE II.

Data for System with Cell Filled with Water. Curve H_2O A.

$M' = 59.990$ gms.

$s = 2.674 \times 10^{-4}$ $t = 20°$ C.

I_v' (Amp/s).	I_v'' (Amp/s).	n' (Head Div. = Cm/k).	n'' (Head Div. = Cm/k).	F_t (Dynes).	H (Gauss).
34.50	79.70	0	100	3.240	2,548
34.70	72.30	"	"	3.917	3,068
34.20	66.25	"	"	4.529	3,594
32.00	58.20	"	"	5.184	4,397
28.40	50.40	"	"	5.479	5,236
23.50	60.90	"	200	5.334	6,160
−16.00	47.40	−200	"	4.245	7,268
−62.80	84.60	−500	500	3.155	7,820
−61.80	73.20	"	"	1.792	8,533
−63.30	59.40	"	"	− 0.675	9,388
−64.90	50.20	"	"	− 2.710	10,010
−68.60	36.90	"	"	− 6.377	10,920
−72.60	25.40	"	"	−10.22	11,750
−75.20	−28.60	"	0	−13.02	12,360
−79.60	−35.20	"	"	−16.83	12,970
−85.00	−43.30	"	"	−22.04	13,810
−90.20	−50.90	"	"	−27.47	14,660
−90.50	−51.40	"	"	−27.92	14,730

TABLE III.

Data for System with Cell Filled with 0.001000 *M* $NiCl_2$ *Solution. Curve* 0.001000 *M* $NiCl_2A$.

$M' = 59.975$ gms.

$s = 2.674 \times 10^{-4}$ $t = 20°$ C.

I_r' (Amp/s).	I_r'' (Amp/s).	n' (Head Div. = Cm/k).	n'' (Head Div. = Cm/k).	F_t (Dynes).	H (Gauss).
35.10	82.10	0	100	3.169	2,450
35.00	70.40	"	"	4.196	3,253
34.10	65.00	"	"	4.683	3,727
31.75	57.30	"	"	5.273	4,507
27.90	49.20	"	"	5.558	5,407
23.30	41.95	"	"	5.302	6,175
18.80	35.50	"	"	4.777	6,896
−47.40	70.80	−400	400	3.359	7,792
−61.50	72.90	−500	500	1.800	8,569
−62.10	68.10	"	"	0.978	8,845
−63.00	58.60	"	"	− 0.768	9,471
−64.50	50.90	"	"	− 2.500	9,979
−67.00	40.20	"	"	− 5.304	10,743
−69.40	33.10	"	"	− 7.514	11,240
−71.90	−23.00	"	0	− 9.980	11,780
−78.20	−34.00	"	"	−12.55	12,250
−78.80	−34.60	"	"	−16.61	13,030
−85.20	−44.10	"	"	−22.77	14,010
−90.80	−52.10	"	"	−28.56	14,880

TABLE IV.

$K \times 10^6$ for Water Computed from Data Read from Curves E_A and H_2O A.

$t = 20°$ C.

H (Gauss).	F_c (Dynes).	F_t (Dynes).	$K \times 10^6$.	Deviation.
3,000	6.725	3.875	−.7469	.00015
4,000	9.933	4.868	.7466	15
5,000	13.35	5.43	.7472	45
6,000	16.75	5.35	.7468	05
7,000	20.08	4.57	.7465	25
8,000	23.13	2.87	.7466	15
9,000	26.15	0.50	.7468	05
10,000	28.95	− 2.70	.7465	25
11,000	31.65	− 6.66	.7467	05
12,000	34.20	−11.40	.7469	15
13,000	36.62	−16.90	.7469	15
14,000	38.98	−23.07	.7466	15
			−.74675	.00017 (0.02%)

TABLE V.

$K \times 10^6$ for 0.001000 M $NiCl_2$ Solution Computed from Data Read from Curves E_A and 0.001000 M $NiCl_2$ A.
t = 20° C.

H (Gauss).	F_e (Dynes).	F_t (Dynes).	$K \times 10^6$.	Deviation.
3,000	6.725	3.891	−.7427	.00031
4,000	9.933	4.900	.7419	49
5,000	13.35	5.48	.7424	01
6,000	16.75	5.42	.7423	09
7,000	20.08	4.65	.7427	31
8,000	23.13	2.98	.7426	21
9,000	26.15	0.65	.7425	11
10,000	28.95	− 2.53	.7424	01
11,000	31.65	− 6.43	.7422	19
12,000	34.20	−11.12	.7423	09
13,000	36.62	−16.57	.7423	09
14,000	38.98	−22.72	.7424	01
			−.74239	.00016 (0.02%)

TABLE VI.

$K \times 10^6$ for Water and Solutions of Nickel and Cobalt Chloride. Summary.
t = 20° C.

Liquid.	Field Range (Gauss $\times 10^{-3}$).	$K \times 10^6$ (Average).	Number of Values of $K \times 10^6$.	Percentage Deviation.
Water (*A*)	3 to 14	− .74675	12	.02
Water (*B*)	"	− .74681	"	.02
		− .74678		
0.001000 *M* $NiCl_2$ solution	"	− .74239	"	.02
0.01000 " " "	"	− .70304	"	.02
0.1000 " " "	"	− .30841	"	.07
1.000 " " "	"	+ 3.6660	"	.04
2.000 " " "	"	+ 8.0333	"	.02
3.765 " " "	3 to 10	+15.7968	8	.02
0.001018 *M* $CoCl_2$ solution	3 to 14	− .73621	12	.03
0.01018 " " "	"	− .64148	"	.03
0.1018 " " "	3 to 12	+ .30534	10	.07
1.0186 " " "	"	+ 9.7819	"	.03
2.0353 " " "	3 to 10	+20.305	8	.04

TABLE VII.

Data for Nickel Chloride Solutions. (Susceptiblity of Air.) Molecular Susceptibility and Magneton Number of Nickel.

$m_s = 129.60$ $t = 20°$ C.

$\kappa_a = 0.02878 \times 10^{-6}$.

Solutions.			$\chi_s \cdot m_s$	$\chi_{Ni} \cdot m_{Ni}$	N	Deviation.
M (mols/liter).	ρ_l (gms/c.c.).	$\kappa_l \times 10^6$				
0.001000	0.99836	− .71361	0.004388	0.004428	16.01	.005
0.01000	0.99946	− .67426	0.004369	0.004409	15.98	25
0.1000	1.01044	− .27963	0.004379	0.004419	16.00	05
1.000	1.11618	3.6948	0.004405	0.004445	16.04	35
2.000	1.22827	8.0621	0.004380	0.004420	16.00	05
3.765	1.42042	15.826	0.004379	0.004419	16.00	05
				0.004423	16.005	.013 (0.08%)

TABLE VIII.

Data for Cobalt Chloride Solutions. (Susceptibility of Air.) Molecular Susceptibility and Magneton Number of Cobalt.

$m_s = 129.89$.

$\kappa_a = 0.02878 \times 10^{-6}$. $t = 20°$ C.

Solutions.			$\chi_s \cdot m_s$	$\chi_{Co} \cdot m_{Co}$	N	Deviation.
M (mols/liter).	ρ_l (gms/c.c.).	$\kappa_l \times 10^6$				
0.001018	0.99825	− .70742	.010300	.010340	24.47	.028
0.01018	0.99939	− .61274	.010330	.010370	24.51	12
0.1018	1.01007	.33412	.010326	.010366	24.50	02
1.0186	1.11364	9.8128	.010325	.010365	24.50	02
2.0353	1.22427	20.326	.010330	.010370	24.51	12
				.010362	24.498	.011 (0.05%)

$m_s\chi_s$ for the most dilute nickel chloride solution should be within about 2 per cent. (giving N to about 1 per cent.), and for the most dilute cobalt solution, within 1 per cent. (giving N to about 0.5 per cent.): the results for the second most dilute nickel solution should hold to about 0.2 per cent. (0.1 per cent. for N), and the results for all other solutions to about 0.1 per cent. (0.05 per cent. for N). These percentages were determined assuming an error of 0.1 per cent. in values of M.

RESULTS AND DISCUSSION.

The principal results obtained from these experiments are:

1. Within a field range of 3,000 to 14,000 gausses for the lower concentrations and of 3,000 to 10,000 for the higher, the magnetic suscepti-

bilities of the liquids used are shown to be independent of the strength of the field.

2. Nickel chloride in aqueous solutions varying in concentration from 0.001000 to 3.765 M and cobalt chloride in concentrations varying from 0.001018 to 2.0353 M have magnetic susceptibilities independent of the concentrations.

3. Under these conditions the molecular susceptibility of nickel at a temperature of 20° C. is found to be 0.004423, and of cobalt 0.010362. The corresponding magneton numbers for nickel and cobalt respectively are 16.0 and 24.5.

4. It follows from (2) that for the solutions used in these experiments the Wiedemann law holds.

Another result obtained in these experiments is that the volume susceptibility of the air of the laboratory at a temperature of 20° C. is, if Piccard's value for water is accepted, 0.0288×10^{-6}: this represents the value for air as regards moisture and pressure for the average conditions obtaining over a period of a number of months.

The susceptibility of air found in these experiments agrees closely (within about 2 per cent.) with Piccard's[1] value given at a temperature of 20° C. and 760 mm. pressure, and (within 0.2 per cent.) with the value recently obtained by Soné.[2]

The values obtained for nickel are in good agreement with the results found by Cabrera, Moles and Guzman, and by Weiss and Mlle. Bruins: the values for cobalt agree with those obtained by Trümpler in his second series of measurements. These results, both for nickel and cobalt, show that in the case of the solutions used no changes in chemical constitution of such a nature as to affect the susceptibility of the salt have accompanied concentration changes. This is not in accordance with the results obtained by Trümpler working with his earlier solutions, or by Cabrera, Jimeno and Marquina for the solutions used by them, or with the results obtained by Quartaroli.

The lack of agreement between the results for cobalt by different observers points to the desirability of a more detailed study of cobalt solutions.

The magneton number for nickel found in these experiments supports the Weiss magneton theory. The number for cobalt presents again the half-magneton already found in the case of other solutions:[3] it would accord with Weiss's theory if the gram-magneton were assigned half the value given in equation (14).

[1] C. R., 155, p. 1234, 1912; Sci. Arch., p. 458, Vol. XXXV, 1913.
[2] Phil. Mag., Ser. 6, p. 309, Vol. 39, 1920.
[3] K. Akad., Amst. Proc., 18, 1915.

Acknowledgments.

The writer takes this opportunity to express her thanks to Professors Fales and Beans for their work on the chemistry of the solutions.

The electromagnet was loaned by the department of physics of Barnard College through the courtesy of Professor Maltby: for this and other courtesies shown by Professor Maltby and Miss Langford of that department the writer wishes to express her appreciation. She is indebted to Mr. Cooey, research mechanician at Fayerweather Hall, both for the construction of apparatus and for suggestions used in improving the design of certain parts. She is most deeply indebted to Professor Albert P. Wills, who suggested the problem and directed the research during the earlier part of the investigation, and to Professor Bergen Davis who assumed the direction of the work during the absence of Professor Wills from the university.

Phoenix Physical Laboratory,
Columbia University.

THE OCCLUSION OF WATER BY GAS-MASK CHARCOAL.

BY BEN E. BROWN.

SYNOPSIS.

Occlusion of Water by Gas-mask Charcoal.—The *variation of amount of water with temperature* was determined between 102° and 140° C. at a pressure of 74 cm. of Hg. About ⅓ of the water came off between 112° and 114° C. The *variation in pressure* with three of the four concentrations, over the range of temperature from 95° to 130° C., was such as to correspond fairly closely to the variation of pressure for solutions.

THE following work was begun with the object of determining the effect of temperature upon the amount of water occluded by charcoal. Some preliminary experiments showed that charcoal, which had been stored in an open tube, gave off steam in unusually large quantities when heated to several degrees above the boiling point of water.

The first part of the work was to determine the relation between amount of water in the charcoal and temperature, at atmospheric pressure. The second part was to determine the relation between pressure and temperature, keeping the water content constant. This relation was determined for various amounts of water. Equilibrium conditions were obtained for all the determinations.

A test tube containing 8.5 grams of gas-mask charcoal, which was obtained through the courtesy of Mr. Van H. Manning, Director of the Bureau of Mines, was flooded with distilled water. It was then placed in a mercury bath and maintained at a constant temperature of from 101° to 104° C. until no more water vapor was driven off. The temperature was then raised a few degrees and again kept constant until equilibrium was obtained. The amount of water driven off was measured by collecting it in a drying tube and weighing. To prevent air from diffusing back into the charcoal a capillary tube of about ¾ mm. bore and 30 cm. in length was sealed to the test bube. The drying tube was connected by means of a ground glass joint, one part of which was sealed to the capillary tube. To prevent water vapor from condensing in the capillary tube a heating coil of German silver wire was wound along its entire length. Moisture from the air was prevented from entering the open end of the drying tube by means of a cotton plug. The thermostat was

actuated by steam pressure and was sensitive to a change of temperature of less than one tenth degree over the range covered.

A temperature range of about 40° was covered in four or five steps. The charcoal was then baked in an electric oven for more than ½ hour at a temperature of about 220° C. The amount of water in the charcoal when the drying tube was first connected was taken as the sum of all the amounts driven off. The amount at the second temperature was this same sum minus the water driven off during the first step, etc.

The results of four complete runs are shown in the curve. The variation of the barometric pressure from 74 cm. of Hg, which was taken as the average barometer reading, was taken into account after the relation between temperature and pressure had been determined. The temperature correction, corresponding to the greatest variation in pressure from 74 cm. was 0.35 of one degree. The amount of water in the charcoal is calculated upon the assumption that all the water was driven off when the charcoal was baked. This is, of course, not accurately true since more water vapor would be obtained if the charcoal was thoroughly outgassed. This small amount of water must be added to all the values given in the tables and curves in order to give the actual amount. Equilibrium was sometimes difficult to obtain, as much as 6 hours being required for points on the steep part of the curve.

For determining the pressure-temperature relation a T-tube was sealed to the charcoal tube. To one branch of the T was sealed a glass tube about 80 cm. in length which was bent to form one side of an ordinary open tube mercury manometer. The remainder of the manometer consisted of a rubber tube and a second glass tube of small bore. The mercury on the side of the manometer next the charcoal was kept at a constant level by raising or lowering the other side of the manometer. By reason of the extra length of the first glass tube the pressure on the rubber connection was always outward, even if the pressure above the charcoal approached zero. This prevented air from leaking into the system through the rubber connection. Again to prevent condensation a heating coil was wound from the charcoal tube to a point well below the top of the mercury column in the manometer. The top of the mercury column was made to coincide with a slit in the heating coil and the height of the opposite column above a fixed point was measured when the second arm of the T was open. The variation of the height of this column when the system was sealed off, together with the barometric reading, gave the pressure of the vapor above the charcoal.

To make a determination, the charcoal was flooded with water through the second branch of the T. The temperature was maintained fixed at

some point well above the boiling point of water. The open tube through which the water was introduced was sealed off before equilibrium had been obtained to make sure that no air diffused into the system. The temperature was then maintained at various points and the pressures measured. The amount of water present was found by noting the temperature which gave a pressure of 74 cm. of Hg and by taking the corresponding point of the first curve.

TABLE I.

Corrected Barometer Reading.	Temp. °C, Reduced to Correspond to Pres. of 74.0 Cm. of Hg.	Weight of Water Driven Off—Mg.	Calculated Weight of Water Retained by Charcoal + What is Retained at 220° C.
73.46	104.1	0	3463
73.40	111.95	1124	2339
73.20	120.2	1934	405
73.20	141.8	290	115
	220 ±	115	
73.07	102.65	0	3482
73.45	110.6	712	2770
73.68	115.6	2034	736
73.68	122.2	384	352
73.68	122.2	3	349
73.73	129	117	232
	220 ±	232	
74.02	101.3	0	3552
73.90	107.0	362	3190
73.77	113.2	1680	1510
73.84	117.3	892	618
	132.2	428	190
	220 ±	190	
73.60	101.35	0	3592
73.65	111.1	837	2755
73.57	115.15	1836	919
	132.2	729	190
	220 ±	190[1]	

For another determination the pressure was raised to something greater than atmospheric pressure. A flame applied to the end of the glass tube allowed it to open up and let out more water. The system was again sealed off and data taken for another curve. Curves 1, 4, 5, were all determined from one initial charge of water. Curves 2 and 3 were each determined by separate initial charges. These two curves happened

[1] The charcoal was in equilibrium at 132.2° C. the second time before baked out. One baking was made for the last two determinations.

to be so close together that they are plotted as one curve. (The points are shown as dots for one curve and as circles for the other.)

To make certain that the pressure measured was due, in measurable extent at least, to water vapor only, the whole system was allowed to cool to room temperature. If the pressure did not fall to a value less than the saturated vapor pressure of water at that temperature a fresh start was made. For curves where the amount of water was small the pressure fell to a lower value than when the amount of water was large. For

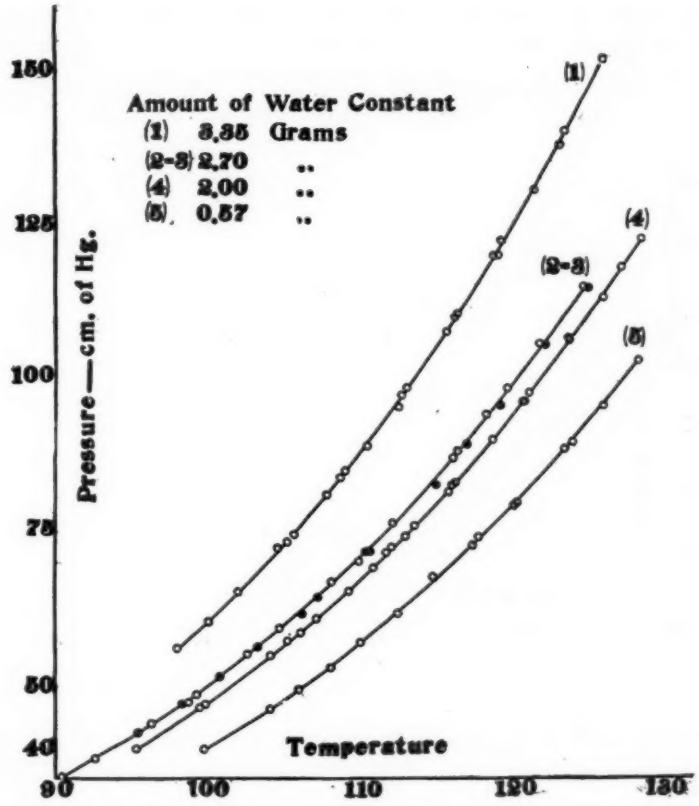


Fig. 1.

curve (5) the value of the pressure was about half the saturated vapor pressure at room temperature. These small pressures could not be measured with sufficient accuracy to insure a small per cent. error, therefore the values obtained were not put into the tables.

The temperatures were read on a mercury thermometer calibrated to tenths of degrees. Owing to temperature gradients in the bath and other causes the error in reading the temperature may be as great as ½ degree. A small amount of mercury distilled from the manometer over into the

charcoal on account of the effects of the heating coil. Since some of the runs extended over two and three days with no noticeable change in pressure at any given temperature it seems evident that this mercury had no appreciable effect on the results obtained. No estimate can be made of the effect upon the data that would be obtained if the charcoal were subjected to an out-gassing process, which is the usual procedure when working with charcoal.

Since charcoal is of a porous nature it seems reasonable to expect that capillarity might play an important part in the occlusion of water. In the third edition of "Properties of Matter" by Poynting and Thomson, page 168, is given a discussion upon the pressure of a vapor in equilibrium

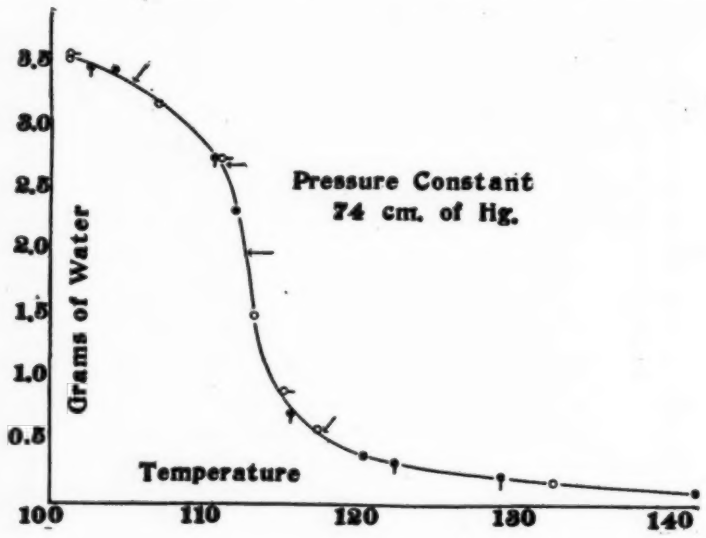


Fig. 2.

with a curved surface of its lqiuid. If a liquid rises to a height h in a capillary tube and the temperature is uniform throughout the system the pressure of the vapor in equilibrium with the top of the thread of liquid must differ from the pressure at the surface of the liquid by an amount equal to the product of the density of the vapor and h, or $\Delta P = -\Delta hd = -K\Delta hP$, assuming that the vapor obeys Boyle's Law. Here d is the density of the vapor and P its pressure. Solving this equation we get

$$-h = \frac{\log P/P_0}{K},$$

P is the pressure of the vapor at the top of the thread of liquid and P_0 is the saturated vapor pressure at the given temperature. If a short

capillary tube is only partly filled with liquid the curvature at each end of the thread of liquid will be the same as it would be at the top of a long capillary in which the liquid rises to its maximum height, and the pressure of vapor in equilibrium with it would be P. If h is calculated from measurements of P, and P_0 is known, the radius r of the capillary may be calculated in terms of the surface tension T and the density of the liquid D.

$$r = \frac{2T}{hDg} = \frac{2TK}{-\log P/P_0},$$

which is essentially the formula used by Lowery and Hulett[1] in calculating the radius of the pores in charcoal upon the assumption that it is a capillary effect which causes water vapor to be absorbed.

TABLE II.

	P	P/P_0	h		P	P/P_0	h
	60.5	.796	3,964		40.8	.621	8,171
	73.2	.807	3,748		47.5	.625	8,114
(1)	88.5	.823	3,441		57.0	.625	8,216
	106.0	.835	3,220	(4)	67.4	.627	8,247
	126.0	.845	3,032		80.2	.631	8,221
	144.0	.852	2,911		94.8	.635	8,197
					107.8	.636	8,168
					121.8	.637	8,260
	43.4	.660	7,127		40.3	.530	1,098
	50.3	.660	7,189		48.0	.529	1,113
	60.2	.664	7,159		57.5	.534	1,108
(2–3)	71.5	.665	7,244	(5)	68.0	.535	1,117
	84.5	.666	7,257		80.1	.537	1,122
	99.7	.668	7,283		91.0	.538	1,127
	113.5	.672	7,227		102.8	.537	1,138

In Table II. are given values of h at various temperatures, with different amounts of water in the charcoal. With rise in temperature, the value of h should decrease if the surface tension decreases as it does under ordinary circumstances. In all but the first curve it is seen that the value of h increases with rise in temperature instead of decreases, as might be expected if the assumption is correct that it is a capillary effect.

It will be noticed in Table II. that for all but curve (1) the values of P/P_0 change very little with change of temperature. It is easily shown from Raoult's Law that the vapor pressure of an ideal solution of fixed concentration divided by the saturated vapor pressure of the solvent at the same temperature is a constant for all temperatures. The closeness

[1] H. H. Lowery and G. A. Hulett, Studies in Adsorbtion of Charcoal, Journal Chem. Soc., July, 1920.

with which charcoal and water follow this law (for solutions) seems to indicate that the molecular forces exerted by charcoal on absorbed substances are similar to the forces between the molecules of a solvent and dissolved substance.

Charcoal[1] has been found to exert an enormous pressure upon a liquid with which its pores has been filled. It seems probable that a vapor within the pores might also be under greater pressure than the free vapor just outside the pores, with which it is in equilibrium. It is possible that data covering a wide range of temperatures and pressures, and using various vapors will throw considerable light upon the nature of the phenomena of occlusion of vapors by charcoal.

P is the pressure of water vapor above charcoal and P_0 is the saturated vapor pressure of water at the same temperature. h is calculated from the formula

$$-h = \frac{\log P/P_0}{K}$$

and is expressed in meters.

In conclusion I wish to express my thanks to Professor F. E. Kester who first proposed the problem to me and who has been closely associated with me during the progress of the work.

BLAKE PHYSICAL LABORATORY,
UNIVERSITY OF KANSAS.

[1] W. D. Harkins, Nat. Acad. Sci. Pro., 6, pp. 49–56, Feb., 1920.

THE LUMINESCENCE OF CERTAIN OXIDES SUBLIMED IN THE ELECTRIC ARC.

BY E. L. NICHOLS AND D. T. WILBER.

SYNOPSIS.

Kathodo Luminescence.—Oxides prepared by sublimation are found to respond to excitation by kathode rays, as well as to flame excitation through a wide range of temperatures.

The Upper Limit of Temperature.—The temperature at which excitation ceases is in general the same, taking each band separately, for flame excitation and for kathodo-excitation.

Color Changes.—The shift in the color of luminescence is in general towards the violet with rising temperature. This shift is produced by the independent growth and decadence of over-lapping bands and not to any lateral movement of the bands themselves.

Influence of Pressure.—Certain bands in the spectra of these oxides are most strongly excited by kathode rays at relatively high pressures, others appear as the vacuum becomes more complete, so that as exhaustion progresses there are marked changes in the color of luminescence.

IN some recent work on luminescence at high temperatures[1] it became necessary to produce thin films of the substances to be investigated. One method was to place a small quantity of the metal, or of one of its salts, in the crater of a direct-current carbon arc and to collect a layer of the finely divided oxide on a metal strip or disk held in the smoke-laden convection current.

The first films thus obtained were of calcium oxide and these under kathode excitation showed the same fine yellow glow already observed in the study of flame excitation and described in the papers just cited. It seemed desirable therefore to determine what other oxides lent themselves to this method of preparation and to learn something of their luminescent properties.

Of the films prepared in the course of our investigation the oxides of the following metals were notably active.

Calcium,	Zirconium,
Magnesium,	Silicon,
Zinc,	Aluminum.

The luminescence of oxides of lead, boron, barium and strontium was very dim. All other oxides tested, including those of cerium, thorium,

[1] Nichols and Wilber, Proc. Nat. Acad. Dec. 1920; PHYSICAL REVIEW (2), 17, 1921, p. 453.

titanium, copper, manganese, bismuth, cadmium and uranium were inactive.

The active films had the following characteristics in common.

1. They are not photo-luminescent at ordinary temperatures.

2. They continue to fluoresce, under kathode bombardment at temperatures far above those at which, in general, photo-luminescence becomes extinct.

3. They are all capable of flame excitation as well as of kathode excitation.

4. In their behavior they are to a great extent independent of the source from which the oxide is obtained: thus calcium oxides produced by the sublimation of the metal, of slaked lime, of calcite crystals or of calcium sulphide appear to be identical as to their luminescence. This is however not a universal characteristic.

5. None of the oxides thus prepared show notable persistent phosphorescence although most of them with small admixtures of chromium, manganese, or bismuth, by suitable heat treatment may, be rendered strikingly phosphorescent. This is particularly true of certain preparations of Al_2O_3 with traces of chromium.

6. Like almost all luminescent substances these oxide films are white or very light in color.

7. With rise of temperature changes in the color of the luminescence occur which correspond in general with those observed by Goldstein,[1] Crookes[2] and by Wiedemann and Schmidt.[3]

Both Goldstein and Crookes noted the change of the kathodo-luminescence of calcium compounds from yellow towards blue on heating. Wiedemann and Schmidt, in a systematic investigation of the effect, showed that the color shift towards violet was common to numerous substances which they tested up to the temperature where the glass of their vacuum tubes began to soften.

THE UPPER TEMPERATURE LIMIT OF KATHODO-LUMINESCENCE.

To estimate the temperature at which luminescence under the excitation of the kathode-rays ceases, a vacuum tube of the form shown in Fig. 1 was prepared. By means of the terminals through the ground glass stopper at *CC* a strip of thin nichrome ribbon (*S*) could be mounted so as to expose the oxide with which it was coated to bombardment from the kathode (*K*). The strip was heated by an electric current and its

[1] Goldstein, Wien. Ber., LXXX, p. 15.

[2] Crookes, Proc. Royal Soc., 1881, p. 209.

[3] Wiedemann and Schmidt, Wiedemann's Annalen, 56, p. 218.

temperature in vacuo was determined in terms of the current. The calibration was made by means of the melting of various selected salts placed in the form of fine powders upon the surface of the strip and these temperatures were checked, above the red heat, by the use of an optical pyrometer (the Morse thermo-gauge).

The extinction of the kathodo-luminescence was quite sharply marked when the nichrome strip with its coating was gradually heated; especially in the case of substances having a greenish or blue fluorescence which was in contrast with the red heat of the background. For calcium oxide, from whatever source sublimed, the temperature of extinction as observed directly with the eye was always the same; *i.e.*, 690° C.

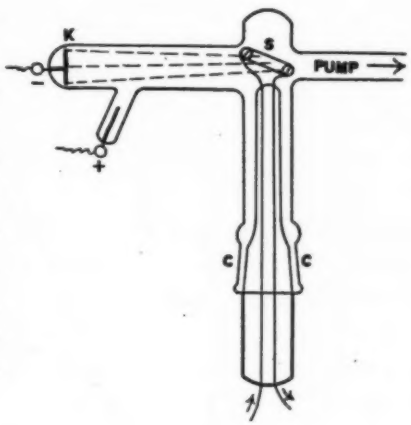


Fig. 1.

By observing the glowing strip with the Morse thermo-gauge we could measure the equivalent "black body" temperatures of the coating at various actual temperatures of the strip.

It had been established in the course of previous studies[1] that the spectrum of the luminescent calcium oxide, like that of the calcites from Franklin Furnace, has two broad, complex, overlapping bands. In our pyrometer readings we therefore used a red screen (equivalent wave-length .65 μ) and a blue green screen (equivalent wave-length .48 μ) and were thus able to study the behavior of the two bands almost independently.

Observations of the color changes showed that the red band began to fall off in brightness at lower temperatures than the green and this was readily confirmed by watching the spectrum as the temperature rose. In spite of the overlapping of the two bands their independent growth and decadence was very striking.

The curves in Fig. 2 indicate roughly, the irregularities being chiefly due to lack of accurate control of the vacuum, the rise of the two curves to a maximum of brightness and their subsidence at the higher temperatures. It will be noted that the red band reaches its maximum at about 280° at which temperature it has more than double the brightness observed at 20° C. The green band is not visible at 20°. It attains its maximum at about 425°. Relatively speaking, red is predominant at

[1] H. L. Howes, PHYSICAL REVIEW (2), 17, 1921, p. 460.

the lower temperatures; green increasingly so from 150° upwards. The pyrometer is apparently more sensitive than the unaided eye in detecting luminescence superimposed upon temperature radiation.

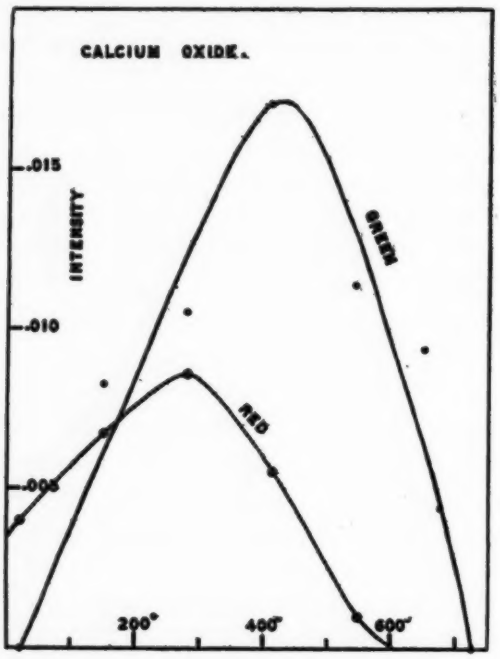


Fig. 2.

TABLE I.

Temperatures and Luminous Intensities of Calcium Oxide.

Red Screen.			Green Screen.		
Temp. of Strip.	Black Body Temp.	Intensity of Luminescence.	Temp. of Strip.	Black Body Temp.	Intensity of Luminescence.
20°	680°	.0040	20°	—	.0000
150°	702°	.0067	150°	712°	.0083
280°	712°	.0083	280°	723°	.0104
410°	695°	.0055	410°	744°	.0170
542°	644°	.0010	542°	726°	.0113
650°	—	—	650°	723°	.0094
675°	666°[1]	.0000	675°	710°	.0044
725°	—	—	725°	690°[1]	.0000

[1] Note that after the disappearance of luminosity the "black body" temperature is less than the indicated temperature of the strip. This is to be expected in the case of a metallic surface coated with a whitish powder.

Thus, the disappearance of the green band is found to lie somewhat above 700° at which temperature it appears to have vanished so far as direct visual observation goes.

Loss of Phosphorescence by Heating.

One of the striking differences between the behavior of CaO. sublimed in the arc and the calcite from which it is derived, is in the loss of phosphorescence.

Calcite from Franklin Furnace was powdered and sifted upon the surface of the nichrome strip; using powdered rosin as a binder.

When cold this preparation glows in the kathode tube with the well known orange-yellow luminescence, not easily distinguishable from that of CaO at the same temperature. It has however persistent phosphorescence of long duration. On exciting at higher temperatures, step by step, we find the phosphorescence continuing up to 475°. Above 185° however the color of the after glow is *white* instead of *red*, and even at lower temperatures the red phosphorescence changes to white during decay as had been noted in an earlier paper.[1]

The white phosphorescence grows rapidly dimmer above 325° but is still persistent being easily observable for several seconds.

Above 475° persistent phosphorescence was not observable. We made no tests for phosphorescence of the vanishing type.

The fluorescence of this preparation ceased at the usual temperature already established for calcium preparations, *i.e.*, 690°. The substance regained its phosphorescent properties on cooling, so long as it was not heated much above this temperature; but on heating for some 15 minutes at 900° it underwent a permanent change—probably loss of CO_2—and became non-phosphorescent even at room temperatures. It was now indistinguishable from our CaO prepared in the arc, its luminescence going over from orange to green-yellow on heating and being quenched at 690°.

Another calcite, taken from an old "Crookes tube" and presumably of European origin, developed a fine pale yellow thermo-luminescence when heated. This began to show at 110°, and increased in intensity up to 240°. At this temperature the orange-yellow fluorescence and the very persistent and strong phosphorescence of 20° were a pale yellow, the phosphorescence dim but persistent. At successively higher temperatures the color of luminescence became paler (*i.e.*, with a movement away from the red). Phosphorescence was very dim at 355°, scarcely visible at 410° and gone at 475°. At 475° the fluorescence was dim, at 542° very dim and no longer discernible at 615°.

[1] Nichols, Howes and Wilber, Physical Review (2), XII., p. 351.

Heating to 900° to convert this specimen to CaO produced a pronounced change in the color of luminescence; the glow when hot being nearly white and when cold of a much paler yellow than before. In this case the power to phosphoresce was not destroyed, there being a rather dim but quite persistent white after glow at the higher temperatures. The ruddy phosphorescence of the calcite however did not reappear.

It seems to be fairly well established that persistent phosphorescence is developed by the presence of considerable proportions of activating materials, as in the phosphorescent sulphides, whereas flame excitation and kathode excitation of *fluorescence* takes place in the case of pure oxides or of preparations containing minute traces only of activating material. To determine the presence or absence of such infinitesimal admixtures as are known to affect the character of kathode-luminescence is of course well nigh impossible.

THE EFFECT OF PRESSURE.

In the course of our experiments we noted that the oxide under observation often began to glow under kathode bombardment at relatively very high pressures, reached its maximum at a moderate vacuum and declined almost to extinction at the lowest pressures reached.

That the luminescence at the higher pressures, where the glass of the tube had not begun to glow was of kathodic excitation and not due to ultra-violet radiation was readily determined by the use of a magnet.

By placing in the same tube a coating of CaO, a crystal of calcite and a synthetic ruby it was found that the oxide was in distinct luminescence during the earlier stages of pumping while the ruby and calcite were still dark.

These, however, which began to glow at somewhat lower pressures, soon greatly exceeded the oxide in brightness and continued to increase as the vacuum improved whereas the oxide reached a maximum of intensity and fell off to relative inactivity. Similar effects were observed with ZrO_2, SiO_2 and Al_2O_3.

To make these observations more definite without going into troublesome refinements a spark gap with nickel-plated balls 25 mm. in diameter was mounted in parallel with the terminals of the tube and observations of the brightness of luminescence of the coating were made with the Morse thermo-gauge, for various sparking distances.

The discharge through the tube during these measurements was obtained from a large four-plate Toepler-Holtz machine.

Any considerable accuracy was difficult owing to the prevailing atmospheric conditions, to unstable states of vacuum within the tube

and the rather rapid decline in the activity of the coatings which were modified and destroyed by the kathode rays. It was found possible, however, to establish in a semi-quantitative manner the relations between tube voltages and luminescence for films of CaO, ZrO_2 and SiO_2. By way of check similar measurements were made upon the kathodo luminescence of a synthetic ruby.

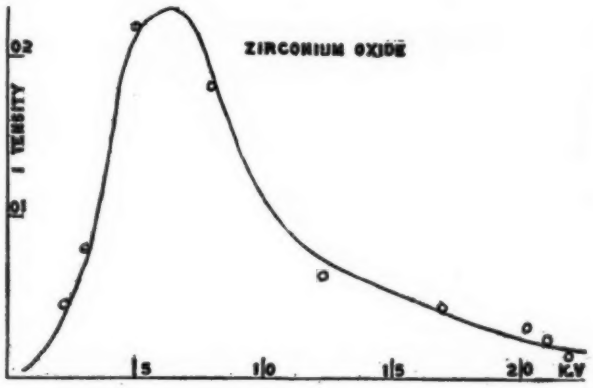


Fig. 3.

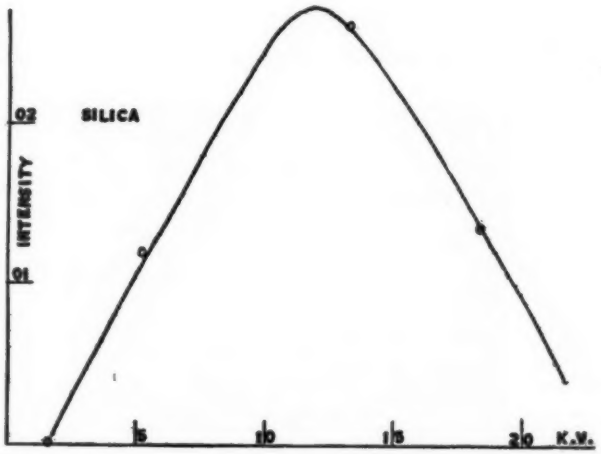


Fig. 4.

The indications of the spark gap were converted into kilovolts by means of curve plotted from measurements by Peek[1] of the relation between spark gap and voltage. The applicability of his data to the conditions of our experiment was verified by the use of a Braun electroscope.

[1] Peek, Dielectric Phenomena, p. 87.

The results of our observations are given in Table II., and Figs. 3, 4 and 5 and, for the ruby, in Table III and Fig. 6.

It appears from these curves (Figs. 3, 4 and 5) that there is a well-marked maximum for each coating beyond which the brightness falls off—

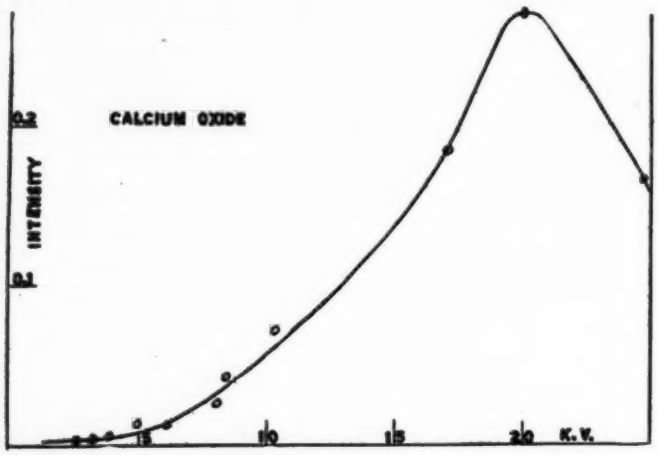


Fig. 5.

a fact which agrees with the visual observations already mentioned. This maximum occurs at the vacuum corresponding to 6 kv. for ZrO_2, 11.5 kv. for SiO_2 and 20 kv. for CaO. Rejuvenation of the glow could

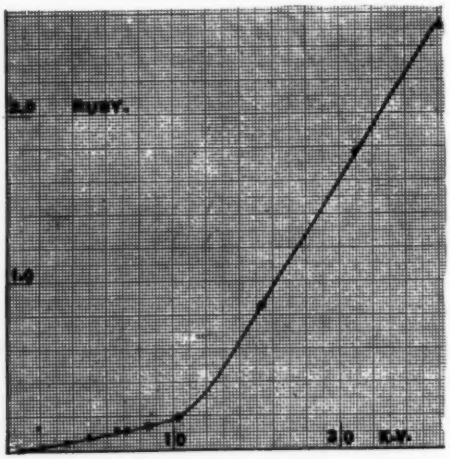


Fig. 6.

be produced at will by pumping until the maximum of brightness was passed and then admitting a small amount of gas to the tube. This brightening up was especially striking in the case of coatings of silicon dioxide.

TABLE II.

Kathodo-luminescence of Three Oxides at Various Tube Voltages.

Substance.	Kilovolts.	B. B. Temp.	Intensity.
Calcium oxide	0.8	620° C.	.0009
	2.1	666°	.0026
	2.6	670°	.0029
	3.1	726°	.0112
	5.0	735°	.0142
	6.6	728°	.0120
	8.5	764°	.0257
	8.9	794°	.0430
	10.4	819°	.0750
	17.3	872°	.1860
	20.2	894°	.2650
	24.9	874°	.1680
	26.2	861°	.1580
Zirconium oxide	2.4	686°	.0046
	3.0	710°	.0080
	5.1	756°	.0220
	8.0	750°	.0182
	12.3	700°	.0061
	17.7	683°	.0043
	20.3	675°	.0034
	20.9	654°	.0025
	21.8	639°	.0015
Silicon oxide	1.4	—	.0000
	5.2	728°	.0120
	13.5	765°	.0262
	18.4	733°	.0135

TABLE III.

Kathodo-luminescence of the Ruby at Various Tube-Voltages.

Kilovolts.	Black Body Temp.	Intensity.
0.8	—	0.
1.7	700°	.0061
2.3	776°	.032
3.6	792°	.045
5.0	834°	.097
6.6	850°	.133
8.5	865°(?)	.140
10.3	944°	.192
15.4	1000°	.841
21.0	—	1.75
26.2	—	2.45

That the bombardment by the kathode particles did not reach a maximum at the highest of the vacua attained in our experiments is clearly indicated in the curve for the synthetic ruby (Fig. 6). The intensity of luminescence of this crystal increased in direct proportion to the tube-voltage above 10 kv., at which point the kathode stream appears to have reached its maximum and constant value.

In view of the fact that these oxides are also capable of flame excitation, one of the most obvious explanations of their luminescence in the vacuum tube would be that the bombardment dislodges portions of the oxide and that it is either the return of these displaced particles that produces fluorescence, or that the material thus partly reduced combines with luminescent effect with free oxygen from the tube. As against the second supposition we found that the rejuvenation of the glow occurred about equally well whether air, pure oxygen or hydrogen was introduced, or whether the pressure change was produced by heating the nichrome strip and driving off its occluded gases or finally by reducing the pressure by the use of liquid air, instead of pumping, and increasing it by the release of the frozen vapors. In this last case especially there would seem to be no change in the free oxygen content of the tube but only change of pressure.

The inference would be that the fluorescence is produced by recombination with oxygen slightly displaced rather than with free oxygen.

That the effect is not due to heating by the kathode particles was very clearly brought out when the heating strip was coated with calcium oxide. The passage of an electric current through the strip enabled us to change the color of the glow of this coating from a ruddy yellow to green before a visible red heat was attained. Kathode bombardment did not however appreciably change the color of the strip, and since the intensity changes with pressure went on at any temperature of the coating, within the temperature range of its activity, it was certain that we had to do with a pressure change rather than a temperature change. In what way the gas pressure affects the actions and reactions, electronic or chemical, upon which the luminescence of these oxides depends remains thus far merely a matter of surmise. It might be suggested, however, as a matter not as yet actually established, that the more intense kathodic bombardments in the higher vacua, tend to drive the oxygen completely off so that there is less and less of the recombination which produces fluorescence. Hence the diminished intensity at higher vacua.

That this effect is not common to kathodo luminescence in general we know from the work of Veazey and T. B. Brown,[1] from Lenard's[2]

[1] T. B. Brown, PHYSICAL REVIEW (2), 11, 1918, pp. 39–57.

[2] Lenard, Annalen der Physik 12, 1903, pp. 449–490.

earlier experiments and from our measurements on the ruby, just described. It is indeed doubtless an effect peculiar to these oxide films and perhaps incident to their mode of preparation.

The case of aluminum oxide in this respect is instructive. This oxide as has been known since the extended researches of Crookes, Urbain, LeCoq de Boisbaudran, and others, exhibits a great variety of colors in the vacuum tube; green, red and blue and the combinations of these. It is a question of impurities and of heat treatment—a matter to be discussed in a subsequent paper. The combination of red and blue occurs frequently, giving rose colors and purples.

We find that when such a specimen is subjected to kathodic bombardment the blue appears first, at the higher pressures and is gradually replaced by red as the vacuum is improved so that we have successively in the same specimen a high pressure glow (blue) and a high vacuum glow (red).

The immediate, tentative explanation, as above, is that the oxygen is more or less loosely attached as the result of different previous heat treatments, etc., so that the pressure relations of the two bands of the fluorescence spectrum are altogether different.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
February, 1921.

ERRATA.

Vol. XVI, July, 1920, in the article by Robert E. Wilson entitled "The Adsorption of Oxygen and Hydrogen by Charcoal," in the second equation on page 13 for $10^2 Td$ read $10^3 Td$.

Vol. XVII, May, 1921, in the article by C. C. Murdock entitled "A Study of the Photo-Active Electrolytic Cell," the last equation on page 643 should read

$$y = 12.0 - \frac{t}{.0844\ t + 1.28}.$$

Index to Volume XVII., Second Series

A.

Abbott. R. B., Damped Electric Oscillations, 482.
Damped Electrical Oscillations, 510.
Absorber, Mass-Absorption Coefficients as a Function of Wave-Length above and below the K X-Ray Limit of the Absorber, *F. K. Richtmyer*, 264.
Absorption, The, of Gamma Rays by Magnetized Iron, *A. H. Compton*, 38.
Absorption, On the, of X-Rays by Chromium, Manganese and Iron, *William Duane and Hugo Fricke*, 529.
Absorption of Light, The, by Mixtures of Inorganic Salts, *E. G. George*, 435.
Absorption Spectra, The, of Ortho-Cresolsulphonphthalein, *W. R. Orndorff, R. C. Gibbs, M. Scott and S. D. Jackson*, 437.
Absorption Spectrum, On the, of HCl, *Walter F. Colby and Charles F. Meyer*, 268.
Absorption, Transmission and, of Sound by a Porous Material, *Paul E. Sabine*, 378.
Acoustics, The, of Large Auditoriums, *C. R. Fountain*, 385.
Acoustical Double Resonator, The, and its Applications, *F. R. Watson*, 380.
Adsorbed Air, The Effect of, on the Ageing of Thin Metal Films, *Lewis R. Koller*, 230.
Ageing, The Effect of Adsorbed Air on the, of Thin Metal Films, *Lewis R. Koller*, 231.
Alloys, X-Ray Analysis of Three Series of, *Mary R. Andrews*, 259.
Alpha Rays, Multiple Valency in the Ionization by, *T. R. Wilkins*, 404.
Alternating Current Permeability, The Effect of a Superposed Constant Field upon the, and Energy Loss in Iron, *Alva W. Smith*, 416.
Aluminum, The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave-Length between 0.13 and 1.05 Angstrom Units in Water, Lithium, Carbon, Nitrogen, Oxygen, and Iron, *C. W. Hewlett*, 284.
American Physical Society:
Abstracts, 230, 371, 508, 521.
Minutes, 228, 367, 508, 519.
Amplification Constant, Calculation of the, of the Weagant Thermionic Vacuum Tube, *Alexander Marcus*, 1.
Analyzer, A Strained Glass Elliptic Polarizer and, *A. DeForest Palmer*, 409.
Anderson, Evald, Nature of Ionization in the Point to Plate Discharge, 516.
Andrews, Mary R., X-Ray Analysis of Three Series of Alloys, 259.
Angle of Incidence, The Effect of, on the Reflection and Secondary Emission of Slow Moving Electrons from Platinum, *John T. Tate*, 394.
Anhysteretic Curves of Iron-Carbon Alloys made with a Braun Tube, *C. W. Waggoner and F. A. Molby*, 427.
Astronomical Measurements, On the Application of Interference Methods to, *A. A. Michelson*, 405.
Atmospheric Electricity. Vertical Electric Currents and the Relation between Terrestrial Magnetism and, *Louis A. Bauer*, 424.
Atom, The Structure of the Helium, *Irving Langmuir*, 339.
Atom, The Stability of the, as Related to Nuclear Composition, *William D. Harkins*, 386.
Atomic Model, A Copernican, Based on Electromagnetic Theory, *Albert C. Crehore*, 541.
Atomic Species, The Abundance of, as Related to Rutherford's Theory of Nuclear Structure, *William D. Harkins*, 388.
Audion Bulbs, Photoelectric Phenomena in Coated Filament, *Ernest Merritt*, 525.
Auditoriums, The Acoustics of Large, *C. R. Fountain*, 385.

B.

Baker, W. C., Effect of Strong Electrostatic Fields on the Vaporization of Tungsten, 239.
Balmer Series, The, of Hydrogen, and the Quantum Theory of Line Spectra, *Raymond T. Birge*, 508.
Balmer Series, The, of Hydrogen and the Quantum Theory of Line Spectra, *Raymond T. Birge*, 589.
Barber, I. Garnett, Secondary Electron Emission from Copper Surfaces, 322.
Secondary Electron Emission from Copper Surfaces, 393.
Barium, New Strontium and, Photo-electric Cells, *Theodore W. Case*, 398.
Barnett, L. J. H., Additional Experiments on the Nature of the Magnetic Molecule, 404.

Barnett, S. J., Additional Experiments on the Nature of the Magnetic Molecule, 404.
Bateman, H., Electricity and Gravitation, 64.
The Lines of Electric Force of a System of Electric Poles Moving along the Same Straight Line, 414.
Bauer, Louis A., Vertical Electric Currents and the Relation between Terrestrial Magnetism and Atmospheric Electricity, 424.
The Eclipse Magnetic Systems of May 29, 1919, 426.
Bayley, P. L., Some Applications of the Method of Images, I., 147.
Binaural Location, The, of Pure Tones, *Thornton G. Fry*, 532.
Birge, Raymond T., The Balmer Series of Hydrogen, and the Quantum Theory of Line Spectra, 508.
The Balmer Series of Hydrogen and the Quantum Theory of Line Spectra, 589.
Bismuth, Magneto-Resistance Effects in Films of, *L. F. Curtiss*, 235.
Blackening, The, of a Photographic Plate as a Function of Intensity of Light and Time of Exposure, *P. S. Helmick*, 135.
Bowen, I. S., The Magnetic Mechanical Analysis of Manganese Steel, 237.
Brant, Laura S., Magnetic Susceptibility of Nickel and Cobalt Chloride Solutions, 678.
Braun Tube, Anhysteretic Curves of Iron-Carbon Alloys made with a, *C. W. Waggoner and F. A. Molby*, 427.
Breakdown Voltage, Conductivity of Insulating Materials near the, *James E. Shrader*, 234.
Breakdown Voltage, Conductivity of Insulating Materials near the, *J. E. Shrader*, 491.
Breit, G., The Distributed Capacity of Inductance Coils, 649.
Bridgman, P. W., The Electrical Resistance of Metals, 161.
A Theory of the Electrical Resistance of Metals, 421.
Brown, Ben E., The Occlusion of Water by Gas-mask Charcoal, 700.
Brown, J. G., A Wave Composition Model, 512.
Brown, S. L., Diathermancy of Ice, Water and Steam, 375.
Bumstead, H. A., 227.

C.

Cady, W. G., The Piezo-Electric Resonator, 531.
Cajori, Florian, Note on the Fahrenheit Scale, 516.
Calcite, The Reflection of X-Rays from, *Bergen Davis and W. M. Stempel*, 526.
Calcite, An Experimental Study of the Reflection of X-Rays from, *Bergen Davis and W. M. Stempel*, 608.
Calcium, The Crystal Structure of, *A. W. Hull*, 42.
Calcium Arc, Pressure Shifts in a, *Henry G. Gale and L. F. Miller*, 428.
Calcium Arc, Some Additional Measurements of Wave-Lengths in the, in Vacuo, *Geo. V. McCauley*, 430.
Capacity, The Distributed, of Inductance Coils, *G. Breit*, 649.
Capillary Flow, The Dynamics of, *Edward W. Washburn*, 273.
Capillary Flow, The Dynamics of, *Edward W. Washburn*, 374.
Capacity, Polarization, and Polarization Resistance as Dependent upon Frequency, *Ernest Merritt*, 524.
Carbon, The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave-Length between 0.13 and 1.05 Angstrom Units in Water, Lithium, Nitrogen, Oxygen, Aluminum, and Iron, *C. W. Hewlett*, 284.
Carbon Dioxide, On the Motion of a Sphere of Oil through, and an Exact Determination of the Coefficient of Viscosity of that Gas by the Oil Drop Method, *L. J. LaSalle*, 354.
Carson, John R., Theory and Calculation of Variable Electrical Systems, 116.
Case, Theodore W., New Strontium and Barium Photo-electric Cells, 398.
Carter, Edna, Character of the Spectra Produced by High Potential Sparks in a Vacuum, 436.
Cathode Ray Oscillograph, A Low Voltage, *J. B. Johnson*, 420.
Cathode Region, Tube Experiments, with the, in a Transverse Magnetic Field, *L. Thompson*, 421.
Charcoal, The Occlusion of Water by Gas-mask, *Ben. E. Brown*, 700.
Child, Clement D., A Continuous Spectrum from Mercury Vapor, 270.
Chromium, On the Absorption of X-Rays by, Manganese and Iron, *William Duane and Hugo Fricke*, 529.
Clark, Harry, A Method of Measuring Surface Tension of Liquids, 537.
Coaxial Cylinders, The Motion of Electrons between, in a Uniform Magnetic Field, *Albert W. Hull*, 539.
Cobalt Chloride Solutions, Magnetic Susceptibility of Nickel and, *Laura S. Brant*, 678.
Coblentz, W. W., The Spectrophotoelectric Sensitivity of Proustite, 245.
Coefficient of Viscosity, On the Motion of a Sphere of Oil through Carbon Dioxide and an Exact Determination of the, of that Gas by the Oil Drop Method, *L. J. LaSalle*, 354.
Colby, Walter F., On the Absorption Spectrum of HCl, 268.
Collisions, Inelastic, of Electrons in Vapors of Certain Compound Molecules, *Paul D. Foote and F. L. Mohler*, 394.

Color Filters for Photographic Uses, *Charles D. Hodgman*, 246.
Compensation Scheme, A, for Electrostatic Measurements, *W. F. G. Swann*, 240.
Compound Molecules, Inelastic Collisions of Electrons in Vapor of Certain, *Paul D. Foote and F. L. Mohler*, 394.
Compton, A. H., The Absorption of Gamma Rays by Magnetized Iron, 38.
Compton, K. T., Note on the Radiating and Ionizing Potentials of Hydrogen, 45.
Conductivity of Insulating Materials near the Breakdown Voltage, *James E. Shrader*, 234.
Conductivity of Insulating Materials near the Breakdown Voltage, *J. E. Shrader*, 491.
Cooling, The Effect of, on a Cement Pier, *Ernest A. Hodgson*, 376.
Copernican Atom, The, Radiating Energy. A Physical Interpretation of Plank's Quantum Rule, *Albert C. Crehore*, 544.
Copernican Atomic Model, A, Based on Electromagnetic Theory, *Albert C. Crehore*, 541.
Copper Surfaces. Secondary Electron Emission from, *I. Garnett Barber*, 322.
Copper Surfaces, Secondary Electron Emission from, *I. Garnett Barber*, 393.
Crehore, Albert C., Comparison between the Fundamental Equations for the Ponderomotive Force for Point Charges due to Larmor-Lorentz and to Megh Nad Saha, 249.
The Newtonian Laws of Gravitation Deduced from the Saha Electromagnetic Theory Applied to the Copernican Atom, 249.
A Copernican Atomic Model Based on Electromagnetic Theory, 541.
The Copernican Atom Radiating Energy, A Physical Interpretation of Planck's Quantum Rule, 544.
Crystals, Reflection of X-Rays from, *W. M. Stempel*, 521.
Crystal Reflection, The Concentration of Monochromatic X-Rays by, *Elmer Dershem and C. T. Dozier*, 519.
Crystal Structure, The, of Ice, *D. M. Dennison*, 20.
Crystal Structure, The, of Calcium, *A. W. Hull*, 42.
Crystal Structure, Graphical Determination of Hexagonal and Tetragonal, from X-Ray Data, *Albert W. Hull and Wheeler P. Davey*, 266.
Crystal Structure, The, of Two Rare Halogen Salts, *Wheeler P. Davey and Frances G. Wick*, 403.
Crystal Structure, The Optical Dispersion and the, of Rock Salt and Sylvite, *H. H. Marvin*, 412.
Crystal Structure, Graphical Determination of Hexagonal and Tetragonal, from X-Ray Data, *Albert W. Hull and Wheeler P. Davey*, 549.
Curtiss, L. F., Magneto-Resistance Effects in Films of Bismuth, 235.

D.

Damped Electric Oscillations, *R. B. Abbott*, 482.
Davey, Wheeler P., Graphical Determination of Hexagonal and Tetragonal Crystal Structure from X-Ray Data, 266.
The Crystal Structure of Two Rare Halogen Salts, 403.
The Cubic Shapes of Certain Ions, as Confirmed by X-Ray Crystal Analysis, 402.
Graphical Determination of Hexagonal and Tetragonal Crystal Structure from X-Ray Data, 549.
Davis, Bergen, The Reflection of X-Rays from Calcite, 526.
An Experimental Study of the Reflection of X-Rays from Calcite, 608.
Davisson, C., The Relation between the Emissive Power of a Metal and its Electrical Resistivity, 261.
Dempster, A. J., Positive Ray Analysis of Magnetism, 427.
Dennison, D. M., The Crystal Structure of Ice, 20.
Density, Influence of the Earth's Potential Gradient upon the Measurement of the Atmospheric Ionic, by the Ebert Ion Counter, *J. F. Mackell*, 390.
Dershem, Elmer, The Concentration of Monochromatic X-Rays by Crystal Reflection, 519.
Differential Thermo-couple, The use of the, in Melting Point Determinations, *O. H. Gish*, 377.
Diurnal Variations in Terrestrial Magnetism and in the Electrical Potential of the Earth, *Fernando Sanford*, 513.
Dodd, L. E., Note on Sheet Striæ as Thin Films in Optical Glass, 270.
Downey, K. Melvina, The Variation of the Residual Ionization in Air with Pressure, 389.
Dozier, C. T., The Concentration of Monochromatic X-Rays by Crystal Reflection, 519.
An Explanation of X-Ray Diffraction Patterns from Rolled Metals, 519.
Duane, William, On the Relative Positions of Lines in X-Ray Spectra, 259.
Approximate Computations of X-Ray Absorption Frequencies, 431.
On the Absorption of X-Rays by Chromium, Manganese and Iron, 529.
Duncan, Jr., R. D., Stability Conditions in Vacuum Tube Circuits, 302.
Dynamics, The, of Capillary Flow, *Edward W. Washburn*, 374.
Dynamic Characteristics, The, of Three Electrode Vacuum Tubes, *John G. Frayne*, 391.

E.

Ebert Ion Counter. Influence of the Earth's Potential Gradient upon the Measure-

ment of the Atmospheric Ionic Density by the Ebert Ion Counter, *J. F. Mackell*, 390.

Eclipse Magnetic Systems, The, of May 29, 1919, *Louis A. Bauer*, 426.

Eckhardt, E. A., A Portable Sound Producer of Considerable Range as to Frequency and Intensity, 380.

The Sound Field of a Parabolic Mirror, 381.

Electron Tube Drive for Tuning Forks, 535.

A High Speed Oscillograph Camera, 536.

Eldridge, A. G., Moving Pictures of Wave Motion, 381.

Electric Arc, The Luminescence of Certain Oxides sublimed in the, *E. L. Nichols and D. T. Wilber*, 707.

Electrostatic Charges, The, of the Earth and Sun and their Relation to Terrestrial Magnetism, *Fernando Sanford*, 243.

Electric Force, The Lines of, of a System of Electric Poles Moving along the Same Straight Line, *H. Bateman*, 414.

Electrical Doublet Theory, An, of the Nature of the Molecular Forces of Chemical and Physical Interaction, *R. D. Kleeman*, 540.

Electrical Resistance, The, of Metals, *P. W. Bridgman*, 161.

Electrical Resistance, A Theory of the, of Metals, *P. W. Bridgman*, 421.

Electrical Systems, Theory and Calculation of Variable, *John R. Carson*, 116.

Electricity and Gravitation, *H. Bateman*, 64.

Electrolytic Cell, The Current from the Photo-active, *C. C. Murdock*, 626, 718.

Electrostatic Fields, Effect of Strong, on the Vaporization of Tungsten, *A. G. Worthing and W. C. Baker*, 239.

Electrostatic Fields, Effect of Strong, on the Vaporization of Tungsten, II., *A. G. Worthing*, 418.

Electrostatic Measurements, A Compensation Scheme for, *W. F. G. Swann*, 240.

Electric Motor, A Thermo, *Paul Kirkpatrick*, 511.

Electrical Oscillations, Damped, *R. B. Abbott*, 510.

Electric Poles, The Lines of Electric Force of a System of, Moving along the same Straight line, *H. Bateman*, 414.

Electrical Potential, Diurnal Variations in Terrestrial Magnetism and in the, of the Earth, *Fernando Sanford*, 513.

Electrical Resistivity, The Relation between the Emissive Power of a Metal and its, *C. Davisson and J. R. Weeks*, 261.

Electrons, Inelastic Collisions of, in Vapors of Certain Compound Molecules, *F. L. Mohler*, 394.

Electrons, The Effect of Angle of Incidence on the Reflection and Secondary Emission of Slow Moving, from Platinum, *John T. Tate*, 394.

Electrons, The Motion of, between Coaxial Cylinders in a Uniform Magnetic Field, *Albert W. Hull*, 539.

Electron Emission, Secondary, from Copper Surfaces, *I. Garnett Barber*, 322.

Electron Emission, Secondary, from Copper Surfaces, *I. Garnett Barber*, 393.

Electron Tube Drive for Tuning Forks, *E. A. Eckhardt, J. C. Karcher and M. Keiser*, 535.

Electromagnetic Induction, Note on, and Relative Motion: A Rejoinder, *W. F. G. Swann*, 242.

Electromagnetic Induction, An Experiment on, and Relative Motion, *W. F. G. Swann*, 423.

Electromagnetic Theory, An, of Gravitation, *H. A. Wilson*, 54.

Electro-Magnetic Theory, The Newtonian Laws of Gravitation Deduced from the Saha, Applied to the Copernican Atom, *Albert C. Crehore*, 252.

Electromagnetic Theory, A Copernican Atomic Model Based on, *Albert C. Crehore*, 541.

Electromotive Forces, A Means of Distinguishing between Intrinsic and Spurious Contact, *R. A. Millikan*, 399.

Elliptic Polarizer, A Strained Glass, and Analyzer, *A. DeForest Palmer*, 409.

Emissive Power of a Metal, The Relation between the, and its Electrical Resistivity, *C. Davisson and J. R. Weeks*, 261.

Emission, The Effect of Angle of Incidence on the Reflection and Secondary, of Slow Moving Electrons from Platinum, *John T. Tate*, 394.

Energy Distribution, Some Peculiarities in, by Speculum Gratings, *L. R. Ingersoll*, 407.

Energy Distribution, Some Peculiarities of Polarization and, by Speculum Gratings, *L. R. Ingersoll*, 491.

Energy Loss, The Effect of a Superposed Constant Field upon the Alternating Current Permeability and, in Iron, *Alva W. Smith*, 416.

Energy Losses in Commercial Hammers, *Edwin Morrison and Robert L. Petry*, 373.

Erikson, Henry A., Size and Aging of Ions Produced in Air, 400.

Errors, A Study of the, Involved in Solving for the Constants of the Fourier's Series by a Calculation Method, *Warren Mason*, 315.

Exposure. The Blackening of a Photographic Plate as a Function of Intensity of Light and Time of, *P. S. Helmick*, 135.

F.

Fahrenheit Scale, Note on the, *Florian Cajori*, 516.

Fant, A. T., The Intensity of Light Transmitted through a deep Slit, 413.
Flame Excitation of Luminescence, *E. L. Nichols and D. T. Wilber*, 269.
Flame Excitation of Luminescence, 453.
Fluid, The Resistance of a Compressible, at High Velocities, *L. Thompson*, 371.
Foote, Paul D., Inelastic Collisions of Electrons in Vapors of Certain Compound Molecules, 394.
Forsythe, W. E., The Current-Temperature Relation for different Pyrometer Filaments, 267.
Found, C. G., Studies with the Ionization Gauge, I., Construction and Method of Calibration, 7.
Fountain, C. R., The Acoustics of Large Auditoriums, 385.
Fourier's Series, A Study of the Errors Involved in Solving for the Constants of, by a Calculation Method, *Warren Mason*, 315.
Franz, F. W., Minimum Sound Energy for Audition, 384.
Frayne, John G., The Dynamic Characteristics of Three Electrode Vacuum Tubes, 391.
A High Frequency Resistance Standard, 415.
Frequency, Polarization Capacity and Polarization Resistance as Dependent upon, *Ernest Merrill*, 524.
Fricke, Hugo, The Spectrum of Helium in the Extreme Ultra-Violet, 434.
On the Absorption of X-Rays by Chromium, Manganese and Iron, 529.
Fry, Thornton C., The Thermionic Current between Parallel Plane Electrodes; Velocities of Emission Distributed according to Maxwell's Law, 441.
The Binaural Location of Pure Tones, 532.

G.

Gale, Henry G., Pressure Shifts in a Calcium Arc, 428.
Gamma Rays, The Absorption of, by Magnetized Iron, *A. H. Compton*, 38.
Gas-mask Charcoal, The Occlusion of Water by, *Ben. E. Brown*, 700.
George E. F., The Absorption of Light by Mixtures of Inorganic Salts, 435.
Gibbs, R. C., The Absorption Spectra of Ortho-Cresolsulphonphthalein, 437.
Gish, O. H., The Use of the Differential Thermo-couple in Melting Point Determinations, 377.
Glass, The Coloration of, by Ultra-violet Light, *J. B. Nathanson*, 408.
Gratings, Some Peculiarities of Polarization and Energy Distribution by Speculum, *L. R. Ingersoll*, 491.
Gravitation, An Electromagnetic Theory of, *H. A. Wilson*, 54.
Gravitation, Electricity and, *H. Bateman*, 64.
Gravitation, The Newtonian Laws of, Deduced from the Saha Electro-magnetic Theory Applied to the Copernican Atom, *Albert C. Crehore*, 252.
Grondahl, L. O., A Method of Studying Sound Waves by Means of a Synchronous Commutator, 258.

H.

Hadfield, Robert, The Magnetic Mechanical Analysis of Manganese Steel, 237.
Hall Coefficient, Constancy of, in Thin Silver Films, *G. W. Stewart*, 232.
Hall Effect, The, and the Nernst Effect in Magnetic Alloys, *Alpheus W. Smith*, 23.
Hall Effect, The, and the Specific Resistance in Silver Films, *G. R. Wait*, 233.
Hall Effect, The, and the Nernst Effect in Magnetic Alloys, *Alpheus W. Smith*, 233.
Halogen Salts, The Crystal Structure of Two Rare, *Wheeler P. Davey and Frances G. Wick*, 403.
Hammers, Energy Losses in Commercial, *Edwin Morrison and Robert L. Petry*, 373.
Harkins, W. D., The Separation of Isotopes, 386.
The Stability of the Atom as Related to Nuclear Composition, 386.
The Abundance of Atomic Species as Related to Rutherford's Theory of Nuclear Structure, 388.
Hartley, R. V. L., The Binaural Location of Pure Tones, 532.
Hebbert, C. M., Some Applications of the Method of Images, II., 157.
Helium Atom, The Structure of the, *Irving Langmuir*, 339.
Helium Atom, The Structures of the, and the Hydrogen Molecule, *Irving Langmuir*, 401.
Helium, The Spectrum of, in the Extreme Ultra-Violet, *Hugo Fricke and Theodore Lyman*, 434.
Helmick, P. S., The Blackening of a Photographic Plate as a Function of Intensity of Light and Time of Exposure, 135.
The Blackening of a Photographic Plate as a Function of Intensity of Light and Time of Exposure, 411.
Hewlett, C. W., A New Tone Generator, 257.
The Absorption and Scattering Coefficients for Homogeneous X-Rays in Several Elements of Low Atomic Weight, 267.
The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave Length between 0.13 and 1.05 Angstrom Units in Water, Lithium, Carbon, Nitrogen, Oxygen, Aluminum, and Iron, 284.
Hexagonal Crystals, A Determination of

Young's Modulus and of the Coefficient of Simple Rigidity of Natural, of Selenium, *L. P. Sieg and R. F. Miller*, 411.
Hidnert, Peter, The Thermal Expansion of Nickel, Monel Metal, Stellite and Stainless Steel, 372.
High Potential Sparks, Character of the Spectra Produced by, in a Vacuum, *Edna Carter*, 436.
Hodgman, Charles D., Color Filters for Photographic Uses, 246.
Hodgson, Ernest A., The Effect of Cooling on a Cement Pier, 376.
Homogeneous X-Rays, The Absorption and Scattering Coefficients for, in Several Elements of Low Atomic Weight, *C. W. Hewlett*, 267.
Homogeneous X-Rays, The Mass Absorption and Mass Scattering Coefficients for, Homogeneous of Wave-Length between 0.13 and 1.05 Angstrom Units in Water, Lithium, Carbon, Nitrogen, Oxygen, Aluminum, and Iron, *C. W. Hewlett*, 284.
Howes, Horace L., The Luminescence of Samarium, 60.
The Spectral Structure of the Luminescence Excited by the Hydrogen Flame, 469.
The Spectral Structure of the Luminescence Excited by the Hydrogen Flame, 538.
Hull, A. W., The Crystal Structure of Calcium, 42.
Graphical Determination of Hexagonal and Tetragonal Crystal Structure from X-Ray Data, 266.
The Motion of Electrons between Coaxial Cylinders in a Uniform Magnetic Field, 539.
Graphical Determination of Hexagonal and Tetragonal Crystal Structure from X-Ray Data, 549.
X-Ray Crystal Analysis of Thirteen Common Metals, 571.
Humidity, Effect of, on Sound Absorption by Plaster Walls, *Paul E. Sabine*, 379.
Hydrogen, Note on the Radiating and Ionizing Potentials of, *K. T. Compton and P. S. Olmstead*, 45.
Hydrogen, The Balmer Series of, and the Quantum Theory of Line Spectra, *Raymond T. Birge*, 508.
Hydrogen, The Balmer Series of, and the Quantum Theory of Line Spectra, *Raymond T. Birge*, 589.
Hydrogen Chloride, On the Absorption Spectrum of, *Walter F. Colby and Charles F. Meyer*, 268.
Hydrogen Flame, The Spectral Structure of the Luminescence Excited by the, *Horace L. Howes*, 469.
Hydrogen Flame, The Spectral Structure of the Luminescence Excited by the, *Horace L. Howes*, 538.
Hydrogen Molecule, The Structures of the Helium Atom and the, *Irving Langmuir*, 401.

I.

Ice, Diathermancy of, Water and Steam, *S. L. Brown*, 375.
Ice, The Crystal Structure of, *D. M. Dennison*, 20.
Images, Some Applications of the Method of, I., *Jacob Kunz and P. L. Bayley*, 147.
Images, Some Applications of the Method of, II., *C. M. Hebbert*, 157.
Inductance Coils, The Distributed Capacity of, *G. Breit*, 649.
Infra-red Spectra of Isotopes, *F. W. Loomis*, 436.
Ingersoll, L. R., Some Peculiarities in Energy Distribution by Speculum Gratings, 407.
Some Peculiarities of Polarization and Energy Distribution by Speculum Gratings, 491.
Index, 719.
Inorganic Salts, The Absorption of Light by Mixtures of, *E. G. George*, 435.
Insulating Materials, Conductivity of, near the Breakdown Voltage, *James E. Shrader*, 234.
Insulating Materials, Conductivity of, near the Breakdown Voltage, *J. E. Shrader*, 491.
Intensity of Light, The Blackening of a Photographic Plate as a Function of, and Time of Exposure, *P. S. Helmick* 135.
Intensity of Light, The Blackening of a Photographic Plate as a Function of, and Time of Exposure, *P. S. Helmick*, 411.
Intensity of Light, The, Transmitted through a Deep Slit, *L. P. Sieg and A. T. Fant*, 413.
Interference Methods, On the Application of, to Astronomical Measurements, *A. A. Michelson*, 405.
Ions, Size and Aging of, Produced in Air, *Henry A. Erikson*, 400.
Ions, The Cubic Shapes of Certain, as Confirmed by X-Ray Crystal Analysis, *Wheeler P. Davey*, 402.
Ions, The Formation of Negative, in Air, *Leonard B. Loeb*, 89.
Ionization, Multiple Valency in the, by Alpha Rays, *T. R. Wilkins*, 404.
Ionization, Nature of, in the Point to Plate Discharge, *Evald Anderson*, 516.
Ionization Gauge, Studies with the, I. Construction and Method of Calibration, *S. Dushman and C. G. Found*, 7.
Ionizing Potentials, Note on the Radiating and, of Hydrogen, *K. T. Compton and P. S. Olmstead*, 45.
Isotopes, The Separation of, *R. S. Millikan*, 386.
Isotopes, Infra-red Spectra of, *F. W. Loomis*, 436.

Iron, The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave-Length between 0.13 and 1.05 Angstrom Units in Water, Lithium, Carbon, Nitrogen, Oxygen, Aluminum and, *C. W. Hewlett*, 284.

Iron: The Effect of a Superposed Constant Field upon the Alternating Current Permeability and Energy Loss in, *Alva W. Smith*, 416.

Iron, On the Absorption of X-Rays by Chromium, Manganese and, *William Duane and Hugo Fricke*, 529.

Iron-Carbon Alloys, Anhysteretic Curves of, made with a Braun Tube, *C. W. Waggoner and F. A. Molby*, 427.

J.

"J" Radiation, The Evidence Regarding the So-called, in the Characteristic X-Ray Spectra of the Elements, *F. K. Richtmyer*, 433.

Jackson, S. D., The Absorption Spectra of Ortho-Cresolsulphonphthalein, 437.

Johnson, J. B., A Low Voltage Cathode Ray Oscillograph, 420.

K.

K X-Ray Limit, Mass-Absorption Coefficients as a Function of Wave-Length above and below the, of the Absorber, *F. K. Richtmyer*, 264.

Kahler, H., The Crystalline Structure of Sputtered Metallic Films, 230.

Karcher, J. C., Apparatus for Measuring Sound Intensity, 382.

A Method of Measuring Phase Differences in a Sound Field, 382.

Electron Tube Drive for Tuning Forks, 535.

Keiser, M., Electron Tube Drive for Tuning Forks, 535.

Kirkpatrick, Paul, A Thermo Electric Motor, 511.

Kleeman, R. D., An Electrical Doublet Theory of the Nature of the Molecular Forces of Chemical and Physical Interaction, 540.

Knipp, Charles T., Note on the Characteristics of the New Singing Tube, 532.

Koller, Lewis R., The Effect of Adsorbed Air on the Ageing of Thin Metal Films, 230.

Koppius, Otto, A Comparison of the Thermionic and Photoelectric Work Function from Platinum, 395.

Kunz, Jacob, Some Applications of the Method of Images, I., 147.

Kurth, E. H., Soft X-Rays of Characteristic Type, 528.

L.

Lamson, K. W., Reflection of Radiation from an Infinite Series of Equally Spaced Planes, 624.

Langmuir, Irving, The Structure of the Helium Atom, 339.

The Structures of the Helium Atom and the Hydrogen Molecule, 401.

Larmor-Lorentz, Comparison between the Fundamental Equations for the Ponderomotive Force for Point Charges due to, and to Megh Nad Saha, *Albert C. Crehore*, 249.

Lassalle, L. J., On the Motion of a Sphere of Oil through Carbon Dioxide and an Exact Determination of the Coefficient of Viscosity of that Gas by the Oil Drop Method, 354.

Lecture Room Wall Charts, *F. J. Rogers*, 509.

Line Spectra, The Balmer Series of Hydrogen, and the Quantum Theory of, *Raymond T. Birge*, 508.

Lithium, The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave-Length between 0.13 and 1.05 Angstrom Units in Water, Carbon, Nitrogen, Aluminum, and Iron, *C. W. Hewlett*, 284.

Loeb, Leonard B., The Formation of Negative Ions in Air, 89.

The Formation of Negative Ions in Air, 400.

Loomis F. W., Infra-red Spectra of Isotopes, 436.

Luminescence, The, of Samarium, *Horace L. Howes*, 60.

Luminescence, Flame Excitation of, *E. L. Nichols and D. T. Wilber*, 269.

Luminescence, The, of Zinc Oxide above the Red Heat, *E. L. Nichols*, 429.

Luminescence, Flame Excitation of, *E. L. Nichols and D. T. Wilber*, 453.

Luminescence, The Spectral Structure of the, Excited by the Hydrogen Flame, *Horace L. Howes*, 469.

Luminescence, The Spectral Structure of the, Excited by the Hydrogen Flame, *Horace L. Howes*, 538.

Luminescence, The, of Certain Oxides sublimed in the Electric Arc, *E. L. Nichols and D. T. Wilber*, 707.

Luminosity, The, of Mercury Vapor Distilled from the Arc in Vacuo, *Norman H. Ricker*, 195.

Lyman, Theodore, The Spectrum of Helium in the Extreme Ultra-Violet, 434.

M.

McCauley, Geo. V., Some Additional Measurements of Wave-Lengths in the Calcium Arc in Vacuo, 430.

Mackell, J. F., Influence of the Earth's Potential Gradient upon the Measurement of the Atmospheric Ionic Density by the Ebert Ion Counter, 390.

Magnesium, Positive Ray Analysis of, *A. J. Dempster*, 427.

Magnetic Alloys, The Hall Effect and the Nernst Effect in, *Alpheus W. Smith*, 23.

Magnetic Alloys, The Hall Effect and the Nernst Effect in, *Alpheus W. Smith*, 233.
Magnetic Field, Tube Experiments, with the Cathode Region in a Transverse, *L. Thompson*, 421.
Magnetic Field, The Motion of Electrons between Coaxial Cylinders in a Uniform, *Albert W. Hull*, 539.
Magnetic Molecule, Additional Experiments on the Nature of the, *S. J. Barnett and L. J. H. Barnett*, 404.
Magnetic Susceptibility of Nickel and Cobalt Chloride Solutions, *Laura S. Brant*, 678.
Magnetized Iron, The Absorption of Gamma Rays by, *A. H. Compton*, 38.
Magneto-Resistance Effects in Films of Bismuth, *L. F. Curtiss*, 235.
Mainsprings of Watches, A Seasonal Breakage of the, *S. R. Williams*, 378.
Manganese, On the Absorption of X-Rays by Chromium, and Iron, *William Duane and Hugo Fricke*, 529.
Manganese Steel, The Magnetic Mechanical Analysis of, Robert Hadfield, *S. R. Williams and I. S. Bowen*, 237.
Marcus, Alexander, Calculation of the Amplification Constant of the Weagant Thermionic Vacuum Tube, 1.
Marvin, H. H., The Optical Dispersion and the Crystal Structure of Rock Salt and Sylvite, 412.
Mason, Warren, A Study of the Errors Involved in Solving for the Constants of Fourier's Series by a Calculation Method, 315.
Mass-Absorption Coefficients as a Function of Wave-Length above and Below the K X-Ray Limit of the Absorber, *F. K. Richtmyer*, 264.
Maxwell's Equations, The Absolute Significance of, *F. D. Murnaghan*, 73.
Measurements of Wave-Lengths, Some Additional, in the Calcium Arc in Vacuo, *Geo. V. McCauley*, 430.
Melting Point Determinations, The use of the Differential Thermo-couple in, *O. H. Gish*, 377.
Mercury Vapor, The Luminosity of, Distilled from the Arc in Vacuo, *Norman H. Ricker*, 195.
Mercury Vapor, A Continuous Spectrum from, *Clement D. Child*, 270.
Merritt, Ernest, Polarization Capacity and Polarization Resistance as Dependent upon Frequency, 524.
Photoelectric Phenomena in Coated Filament Audion Bulbs, 525.
Metals, X-Ray Crystal Analysis of Thirteen Common, *Albert W. Hull*, 571.
Metal Films, The Effect of Adsorbed Air on the Ageing of Thin, *Lewis R. Koller*, 231.
Metallic Films, The Crystalline Structure of Sputtered, *H. Kahler*, 230.
Metals, A Theory of the Electrical Resistance of, *P. W. Bridgman*, 421.
Metals, An Explanation of X-Ray Diffraction Patterns from, *C. T. Dozier*, 519.
Meyer, Charles F., On the Absorption Spectrum of HCl, 268,
Michelson, A. A., On the Application of Interference Methods to Astronomical Measurements, 405.
Miller, Dayton C., On the Propagation of the Sound Wave from the Muzzle of a Large Gun, 255.
Miller, L. F., Pressure Shifts in a Calcium Arc, 428.
Miller, R. F., A Determination of Young's Modulus and of the Coefficient of Simple Rigidity of Natural Hexagonal Crystals of Selenium, 411.
Millikan, R. A., A Means of Distinguishing between Intrinsic and Spurious Contact Electromotive Forces, 399.
Mohler, F. L., Inelastic Collisions of Electrons in Vapors of Certain Compound Molecules, 394.
Molby, F. A., Anhysteretic Curves of Iron-Carbon Alloys made with a Braun Tube, 427.
Molecular Forces, An Electrical Doublet Theory of the Nature of the, of Chemical and Physical Interaction, *R. D. Kleeman*, 540.
Monel Metal, The Thermal Expansion of Nickel, Stellite and Stainless Steel, *William Souder and Peter Hidnert*, 372.
Monochromatic X-Rays, The Concentration of, by Crystal Reflection, *Elmer Dershem and C. T. Dozier*, 519.
Morrison, Edwin, Energy Losses in Commercial Hammers, 373.
Multiple Valency in the Ionization by Alpha Rays, *T. R. Wilkins*, 404.
Murdock, C. C., The Current from the Photo-active Electrolytic, Cell, 626, 718.
Mulliken, R. S., The Separation of Isotopes, 386.
Murnaghan, F. D., The Absolute Significance of Maxwell's Equations, 73.

N.

Náthanson, J. B., The Coloration of Glass by Ultra-violet Light, 408.
Negative Ions, The Formation of, in Air, *Leonard B. Loeb*, 400.
Nernst Effect, The Hall Effect and the, in Magnetic Alloys, *Alpheus W. Smith*, 23.
Nernst Effect, The Hall Effect and the, in Magnetic Alloys, *Alpheus W. Smith*, 233.
New Books, 70, 272, 438, 648.
Newtonian Laws, The, of Gravitation deduced from the Saha Electro-magnetic Theory Applied to the Copernican Atom, *Albert C. Crehore*, 249.
Nitrogen, The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave-Length

between 0.13 and 1.05 Angstrom Units in Water, Lithium, Carbon, Oxygen, Aluminum, and Iron, *C. W. Hewlett*, 284.

Nichols, E. L., Flame Excitation of Luminescence, 269.

The Luminescence of Zinc Oxide above the Red Heat, 429.

Flame Excitation of Luminescence, 453.

The Luminescence of Certain Oxides sublimed in the Electric Arc, 707.

Nickel, The Thermal Expansion of, Monel Metal, Stellite and Stainless Steel, *William Souder and Peter Hidnert*, 372.

Nickel, Magnetic Susceptibility of, and Cobalt Chloride Solutions, *Laura S. Brant*, 678.

Nuclear Composition, The Stability of the Atom as Related to, *William D. Harkins*, 386.

Nuclear Structure: The Abundance of Atomic Species as Related to Rutherford's Theory of, *William D. Harkins*, 388.

O.

Occlusion of Water, The, by Gas-mask Charcoal, *Ben. E. Brown*, 700.

Oil Drop Method. On the Motion of a Sphere of Oil through Carbon Dioxide and an Exact Determination of the Coefficient of Viscosity of that Gas by the, *L. J. LaSalle*, 354.

Olmstead, P. S., Note on the Radiating and Ionizing Potentials of Hydrogen, 45.

Optical Dispersion, The, and the Crystal Structure of Rock Salt and Sylvite, *H. H. Marvin*, 412.

Optical Glass, Note on Sheet Striæ as Thin Films in, *L. E. Dodd*, 270.

Orndorff, W. R., The Absorption Spectra of Ortho-Cresolsulphonphthalein, 437.

Ortho-Cresolsulphonphthalein, The Absorption Spectra of, *W. R. Orndorff, R. C. Gibbs, M. Scott and S. D. Jackson*, 437.

Oscillations, Damped Electric, *R. B. Abbott*, 482.

Oscillograph Camera, A High Speed, *E. A. Eckhardt*, 536.

Oxides, The Luminescence of Certain, sublimed in the Electric Arc, *E. L. Nichols and D. T. Wilber*, 707.

Oxygen, The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave-Length between 0.13 and 1.05 Angstrom Units in Water, Lithium, Carbon, Nitrogen, Aluminum, and Iron, *C. W. Hewlett*, 284.

P.

Palmer, A. DeForest, A Strained Glass Elliptic Polarizer and Analyzer, 409.

Parabolic Mirror, The Sound Field of a, *E. A. Eckhardt*, 381.

Patterson, R. A., On the Relative Positions of Lines in X-Ray Spectra, 259.

Petry, Robert L., Energy Losses in Commercial Hammers, 373.

Phase Differences, A Method of Measuring, in a Sound Field, *J. C. Karcher*, 382.

Photo-active Electrolytic Cell, The Current from the, *C. C. Murdock*, 626, 718.

Photo-electric Cells, New Strontium and Barium, *Theodore W. Case*, 398.

Photoelectric Phenomena in Coated Filament Audion Bulbs, *Ernest Merritt*, 525.

Photoelectric Work Function, A Comparison of the Thermionic and from Platinum, *Otto Koppius*, 395.

Photographic Plate, The Blackening of a, as a Function of Intensity of Light and Time of Exposure, *P. S. Helmick*, 135.

Photographic Plate, The Blackening of a, as a Function of Intensity of Light and Time of Exposure, *P. S. Helmick*, 411.

Piezo-Electric Activity of Rochelle Salt under Various Conditions, *J. Valasek*, 422.

Piezo-Electric and Allied Phenomena in Rochelle Salt, *J. Valasek*, 475.

Piezo-Electric Resonator, The, *W. G. Cady*, 531.

Planck's Quantum Rule, A Physical Interpretation of, *Albert C. Crehore*, 544.

Plate Discharge, Nature of Ionization in the Point to, *Evald Anderson*, 516.

Platinum, The Effect of Angle of Incidence on the Reflection and Secondary Emission of Slow Moving Electrons from, *John T. Tate*, 394.

Platinum. A Comparison of the Thermionic and Photoelectric Work Function from Platinum, 395.

Polarization, Some Peculiarities of, and Energy Distribution by Speculum Gratings, *L. R. Ingersoll*, 491.

Polarization Capacity and Polarization Resistance as Dependent upon Frequency, *Ernest Merritt*, 524.

Pondero-motive Force, Comparison between the Fundamental Equations for the, for Point Charges due to Larmor-Lorentz and to Megh Nad Saha, *Albert C. Crehore*, 249.

Potential Gradient, Influence of the Earth's, upon the Measurement of the Atmospheric Ionic Density by the Ebert Ion Counter, *J. F. Mackell*, 390.

Proustite, The Spectrophotoelectric Sensitivity of, *W. W. Coblentz*, 245.

Pure Tones, The Binaural Location of, *R. V. L. Hartley and Thornton G. Fry*, 532.

Pyrometer Filaments, The Current-Temperature Relation for different, *W. E. Forsythe*, 267.

Q.

Quantum Rule, A Physical Interpretation of Planck's, *Albert C. Crehore*, 544.

Quantum Theory, The Balmer Series of Hydrogen and the, of Line Spectra, *Raymond T. Birge*, 589.

R.

Radiation, Reflection of, from an Infinite Series of Equally Spaced Planes, *K. W. Lamson*, 624.

Radiating Energy, The Copernican Atom, A Physical Interpretation of Planck's Quantum Rule, *Albert C. Crehore*, 544.

Radiating Potentials, Note on the, and Ionizing Potentials of Hydrogen, *K. T. Compton and P. S. Olmstead*, 45.

Ray Analysis, Positive, of Magnesium, *A. J. Dempster*, 427.

Red Heat, The Luminescence of Zinc Oxide above the Red Heat, *E. L. Nichols*, 429.

Reed Instrument, Broken Tone from, *John B. Taylor*, 534.

Reflection of X-Rays, The, from Calcite, *Bergen Davis and W. M. Stempel*, 526.

Relative Motion. Note on Electromagnetic Induction and, A Rejoinder, *W. F. G. Swann*, 242.

Relative Motion, An Experiment on Electromagnetic Induction and, *W. F. G. Swann*, 423.

Residual Ionization, The Variation of the, in Air with Pressure, *K. Melvina Downey*, 389.

Resistance, The, of a Compressible Fluid at High Velocities, *L. Thompson*, 371.

Resistance, The Electrical, of Metals, *P. W. Bridgman*, 161.

Resistance, Polarization Capacity and Polarization, as Dependent upon Frequency, *Ernest Merritt*, 524.

Resistance Standard, A High Frequency, *John G. Frayne*, 415.

Resonator, The Acoustical Double, and its Applications, *F. R. Watson*, 380.

Resonator, The Piezo-Electric, *W. G. Cady*, 531.

Richtmyer, F. K., Mass-Absorption Coefficients as a Function of Wave-Length above and below the K X-Ray Limit of the Absorber, 264.

The Evidence Regarding the So-called "J" Radiation in the Characteristic X-Ray Spectra of the Elements, 433.

Ricker, Norman H., The Luminosity of Mercury Vapor Distilled from the Arc in Vacuo, 195.

Rigidity, A Determination of Young's Modulus and of the Coefficient of Simple, of Natural Hexagonal Crystals of Selenium, *L. P. Sieg and R. F. Miller*, 411.

Rochelle Salt, Piezo-electric Activity of, under Various Conditions, *J. Valasek*, 422.

Rochelle Salt, Piezo-Electric and Allied Phenomena in, *J. Valasek*, 475.

Rock Salt, The Optical Dispersion and the Crystal Structure of, and Sylvite, *H. H. Marvin*, 412.

Rogers, F. J., Lecture Room Wall Charts, 509.

Rutherford's Theory, The Abundance of Atomic Species as Related to, of Nuclear Structure, *William D. Harkins*, 388.

S.

Sabine, Paul E., Transmission and Absorption of Sound by a Porous Material, 378.

Sabine, Paul E., Effect of Humidity on Sound Absorption by Plaster Walls, 379.

Saha, Megh Nad, Comparison between the Fundamental Equations for the Ponderomotive Force for Point Charges due to Larmor-Lorentz and to, *Albert C. Crehore*, 249.

Samarium, The Luminescence of, *Horace L. Howes*, 60.

Sanford, Fernando, The Electrostatic Charges of the Earth and Sun and their Relation to Terrestrial Magnetism, 243.

Diurnal Variations in Terrestrial Magnetism and in the Electrical Potential of the Earth, 513.

Sun Spots and Terrestrial Magnetic Storms, 515.

Scott, M., The Absorption Spectra of Ortho-Cresolsulphonphthalein, 437.

Selenium, A Determination of Young's Modulus and of the Coefficient of Simple Rigidity of Natural Hexagonal Crystals of, *L. P. Sieg and R. F. Miller*, 411.

Sensitivity, The Spectrophotoelectric, of Proustite, *W. W. Coblentz*, 245.

Sheet Striæ, Note on, as Thin Films in Optical Glass, *L. E. Dodd*, 270.

Shrader, James E., Conductivity of Insulating Materials near the Breakdown Voltage, 234.

Conductivity of Insulating Materials near the Breakdown Voltage, 491.

Sieg, L. P., A Determination of Young's Modulus and of the Coefficient of Simple Rigidity of Natural Hexagonal Crystals of Selenium, 411.

The Intensity of Light Transmitted through a Deep Slit, 413.

Silver Films, Constancy of Hall Coefficient in Thin, *G. W. Stewart*, 232.

Silver Films, The Hall Effect and the Specific Resistance in, *G. R. Wait*, 233.

Singing Tube, Note on the Characteristics of the New, *Charles T. Knipp*, 532.

Slit, The Intensity of Light Transmitted through a Deep, *L. P. Sieg and A. T. Fant*, 413.
Smith, Alpheus W., The Hall Effect and the Nernst Effect in Magnetic Alloys, 23.
The Hall Effect and the Nernst Effect in Magnetic Alloys, 233.
Smith, Alva W., The Effect of a Superposed Constant Field upon the Alternating Current Permeability and Energy Loss in Iron, 416.
Souder, William, The Thermal Expansion of Nickel, Monel Metal, Stellite and Stainless Steel, 372.
Sound Absorption, Effect of Humidity on, by Plaster Walls, *Paul E. Sabine*, 379.
Sound Energy, Minimum, for Audition, *F. W. Franz*, 384.
Sound Field, The, of a Parabolic Mirror, *E. A. Eckhardt*, 381.
Sound Field, A Method of Measuring Phase Differences in a, *J. C. Karcher*, 382.
Sound Intensity, Apparatus for Measuring, *J. C. Karcher*, 382.
Sound Producer, A Portable, of Considerable Range as to Frequency and Intensity, *E. A. Eckhardt*, 380.
Sound, Transmission and Absorption of, by a Porous Material, *Paul E. Sabine*, 378.
Sound Wave, On the Propagation of the, from the Muzzle of a Large Gun, *Dayton C. Miller*, 255.
Sound Waves, A Method of Studying, by Means of a Synchronous Commutator, *L. O. Grandahl*, 258.
Spaced Planes, Reflection of Radiation from an Infinite Series of, *K. W. Lamson*, 624.
Specific Resistance, The Hall Effect and the, in Silver Films, *G. R. Wait*, 233.
Speculum Gratings, Some Peculiarities in Energy Distribution by Speculum Gratings, *L. R. Ingersoll*, 407.
Speculum Gratings, Some Peculiarities of Polarization and Energy Distribution by, *L. R. Ingersoll*, 491.
Spectral Structure, The, of the Luminescence Excited by the Hydrogen Flame, *Horace L. Howes*, 538.
Sphere of Oil, On the Motion of a, through Carbon Dioxide and an Exact Determination of the Coefficient of Viscosity of that Gas by the Oil Drop Method, *L. J. Lasalle*, 354.
Spectra, Character of the, Produced by High Potential Sparks in a Vacuum, *Edna Carter*, 436.
Spectra. The Balmer Series of Hydrogen and the Quantum Theory of Line, *Raymond T. Birge*, 589.
Spectrum, A Continuous, from Mercury Vapor, *Clement D. Child*, 270.
Stability Conditions in Vacuum Tube Circuits, *R. D. Duncan, Jr.*, 302.
Stainless Steel, The Thermal Expansion of Nickel, Monel Metal, Stellite and, *William Souder and Peter Hidnert*, 372.
Steam, Diathermancy of Ice, Water and, *S. L. Brown*, 375.
Stellite, The Thermal Expansion of Nickel, Monel Metal, and Stainless Steel, *William Souder and Peter Hidnert*, 372.
Stempel, W. M., Reflection of X-Rays from Crystals, 521.
The Reflection of X-Rays from Calcite, 526.
An Experimental Study of the Reflection of X-Rays from Calcite, 608.
Stewart, G. W., Constancy of Hall Coefficient in Thin Silver Films, 232.
An Acoustic Wave Filter, 382.
Strontium, New, and Barium Photo-electric Cells, *Theodore W. Case*, 398.
Sun Spots and Terrestrial Magnetic Storms, *Fernando Sanford*, 515.
Surface Tension, A Method of Measuring, of Liquids, *Harry Clark*, 537.
Swann, W. F. G., A Compensation Scheme for Electrostatic Measurements, 240.
Swann, W. F. G., Note on Electromagnetic Induction and Relative Motion: A Rejoinder, 242.
An Experiment on Electromagnetic Induction and Relative Motion, 423.
Sylvite, The Optical Dispersion and the Crystal Structure of Rock Salt and, *H. H. Marvin*, 412.
Synchronous Commutator, A Method of Studying Sound Waves by Means of a, *L. O. Grondahl*, 258.

T.

Tate, John T., The Effect of Angle of Incidence on the Reflection and Secondary Emission of Slow Moving Electrons from Platinum, 394.
Taylor, John B., Broken Tone from Reed Instrument, 534.
Terrestrial Magnetism, The Electrostatic Charges of the Earth and Sun and Their Relation to, *Fernando Sanford*, 243.
Terrestrial Magnetism, Vertical Electric Currents and the Relation between, and Atmospheric Electricity, *Louis A. Bauer*, 424.
Terrestrial Magnetism, Diurnal Variations in, and in the Electrical Potential of the Earth, *Fernando Sanford*, 513.
Terrestrial Magnetic Storms, Sun Spots and, *Fernando Sanford*, 515.
Thermal Expansion, The, of Nickel, Monel Metal, Stellite and Stainless Steel, *William Souder and Peter Hidnert*, 372.
Thermionic Current, The, between Parallel Plane Electrodes; Velocities of Emis-

sion Distributed according to Maxwell's Law, *Thornton C. Fry*, 441.
Thermionic Discharges, Pulsating, in Evacuated Tungsten Lamps, *A. G. Worthing*, 237.
Thermionic Work Function, A Comparison of the, Photoelectric and, from Platinum, *Otto Koppius*, 395.
Thompson, L., The Resistance of a Compressible Fluid at High Velocities, 371.
Tube Experiments, with the Cathode Region in a Transverse Magnetic Field, 421.
Time of Exposure, The Blackening of a Photographic Plate as a Function of Intensity of Light and, *P. S. Helmick*, 411.
Tone, Broken, from Reed Instrument, *John B. Taylor*, 534.
Tone Generator, A New, *C. W. Hewlett*, 257.
Transmission and Absorption of Sound by a Porous Material, *Paul E. Sabine*, 378.
Tube Experiments, with the Cathode Region in a Transverse Magnetic Field, *L. Thompson*, 421.
Tungsten, Effect of Strong Electrostatic Fields on the Vaporization of, *A. G. Worthing and W. C. Baker*, 239.
Tungsten Lamps, Pulsating Thermionic Discharges in Evacuated, *A. G. Worthing*, 237.
Tuning Forks, Electron Tube Drive for, *E. A. Eckhardt, J. C. Karcher and M. Keiser*, 535.

U.

Ultra-Violet, The Spectrum of Helium in the Extreme, *Hugo Fricke and Theodore Lyman*, 434.
Ultra-violet Light, The Coloration of Glass by, *J. B. Nathanson*, 408.

V.

Vacuum Tube, Calculation of the Amplification Constant of the Weagant Thermionic, *Alexander Marcus*, 1.
Vacuum Tubes, The Dynamic Characteristics of Three Electrode, *John G. Frayne*, 391.
Vacuum Tube Circuits, Stability Conditions in, *R. D. Duncan, Jr.*, 302.
Valasek, J., Piezo-electric Activity of Rochelle Salt under Various Conditions, 422.
Piezo-Electric and Allied Phenomena in Rochelle Salt, 475.
Vaporization of Tungsten, Effect of Strong Electrostatic Fields on the, *A. G. Worthing and W. C. Baker*, 239.
Vaporization of Tungsten, II., Effect of Strong Electrostatic Fields on the, *A. G. Worthing*, 418.
Vertical Electric Currents and the Relation between Terrestrial Magnetism and Atmospheric Electricity, *Louis A. Bauer*, 424.

W.

Waggoner, C. W., Anhysteretic Curves of Iron-Carbon Alloys made with a Braun Tube, 427.
Wait, G. R., The Hall Effect and the Specific Resistance in Silver Films, 233.
Wall Charts, Lecture Room, *F. J. Rogers*, 509.
Washburn, Edward W., The Dynamics of Capillary Flow, 273.
The Dynamics of Capillary Flow, 374.
Water, The Mass Absorption and Mass Scattering Coefficients for Homogeneous X-Rays of Wave-Length between 0.13 and 1.05 Angstrom Units in, Lithium, Carbon, Nitrogen, Oxygen, Aluminum, and Iron, *C. W. Hewlett*, 284.
Water, Diathermancy of Ice, and Steam, *S. L. Brown*, 375.
Watson, F. R., The Acoustical Double Resonator and its Applications, 380.
Moving Pictures of Wave Motion, 381.
Wave Composition Model, A, *J. G. Brown*, 512.
Wave Filter, An Acoustic, *G. W. Stewart*, 382.
Wave Motion, Moving Pictures of, *F. R. Watson*, 381.
Weagant Thermionic Vacuum Tube, Calculation of the Amplification Constant of the, *Alexander Marcus*, 1.
Weeks, J. R., The Relation between the Emissive Power of a Metal and its Electrical Resistivity, 261.
Wick, Frances G., The Crystal Structure of Two Rare Halogen Salts, 403.
Wilber, D. T., Flame Excitation of Luminescence, 269.
Flame Excitation of Luminescence, 453.
The Luminescence of Certain Oxides sublimed in the Electric Arc, 707.
Williams, S. R., The Magnetic Mechanical Analysis of Manganese Steel, 237.
A Seasonal Breakage of the Mainsprings of Watches, 378.
Wilkins, T. R., Multiple Valency in the Ionization by Alpha Rays, 404.
Wilson, H. A., An Electromagnetic Theory of Gravitation, 54.
Worthing, A. G., Pulsating Thermionic Discharges in Evacuated Tungsten Lamps, 237.
Effect of Strong Electrostatic Fields on the Vaporization of Tungsten, 239.
Effect of Strong Electrostatic Fields on the Vaporization of Tungsten, II., 418.

X.

X-Ray Absorption Frequencies, Approximate Computations of, *William Duane*, 431.

X-Ray Analysis of Three Series of Alloys, *Mary R. Andrews*, 259.
X-Ray Crystal Analysis, The Cubic Shapes of Certain Ions, as Confirmed by, *Wheeler P. Davey*, 402.
X-Ray Crystal Analysis of Thirteen Common Metals, *Albert W. Hull*, 571.
X-Ray Data, Graphical Determination of Hexagonal and Tetragonal Crystal Structure from, *Albert W. Hull and Wheeler P. Davey*, 266.
X-Ray Data, Graphical Determination of Hexagonal and Tetragonal Crystal Structure from, *Albert W. Hull and Wheeler P. Davey*, 549.
X-Ray Diffraction Patterns, An Explanation of, from Rolled Metals, *C. T. Dozier*, 519.
X-Rays, On the Absorption of, by Chromium, Manganese and Iron, *William Duane and Hugo Fricke*, 529.
X-Rays, Reflection of, from Crystals, *W. M. Stempel*, 521.
X-Rays, The Reflection of, from Calcite, *Bergen Davis and W. M. Stempel*, 526.
X-Rays, An Experimental Study of the Reflection of, from Calcite, *Bergen Davis and W. M. Stempel*, 608.
X-Rays, Soft, of Characteristic Type, *E. H. Kurth*, 528.
X-Ray Spectra, On the Relative Positions of Lines in, *William Duane and R. A. Patterson*, 259.
X-Ray Spectra, The Evidence Regarding the So-called "J" Radiation in the Characteristic, of the Elements, *F. K. Richtmyer*, 433.
X-Rays, The Concentration of Monochromatic, by Crystal Reflection, *Elmer Dershem and C. T. Dozier*, 519.

Y.

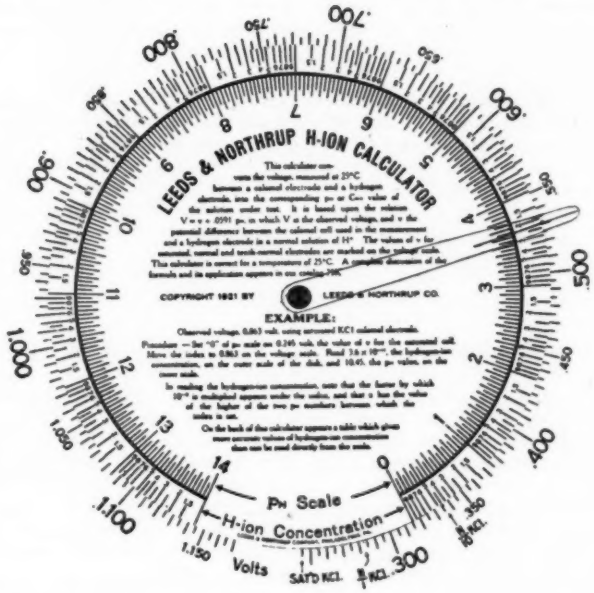
Young's Modulus, A Determination of, and of the Coefficient of Simple Rigidity of Natural Hexagonal Crystals of Selenium, *L. P. Sieg and R. F. Miller*, 411.

Z.

Zinc Oxide, The Luminescence of, above the Red Heat, *E. L. Nichols*, 429.

H-ION CALCULATOR

CONVENIENT—QUICK—ACCURATE



In making a determination of H-ion concentration of a solution by the electrometric method, the actual Measurement is one of potential difference, or voltage between a hydrogen and a calomel electrode in contact with the solution. The H-ion concentration, or the p_H value, must then be found by reference to conversion tables or by calculation from an equation.

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All correspondence relating to contributions should be addressed to THE PHYSICAL REVIEW, Ithaca, N. Y. Manuscripts will be acknowledged as soon as received. Articles submitted for publication in the REVIEW are not to be submitted for publication elsewhere unless the authorization of the editors has been obtained, and proper reference to the REVIEW is made. Manuscript should be ready for the printer; the editors cannot assume responsibility for its correctness. The presentation should be as concise as is consistent with clearness; all unnecessary duplication of data in table and in curves is to be avoided, and tabular matter should be introduced *in extenso* only when the exact numerical values affect the value of the paper. It is requested that the metric system be used in all cases. Whenever the value of a quantity is expressed in any other system, its value in the metric system should be added in parenthesis. *Every article is to be preceded by a synopsis;* see following page.

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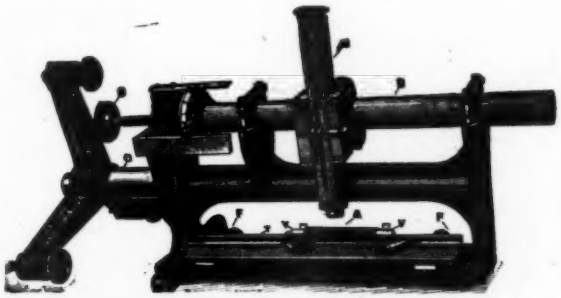

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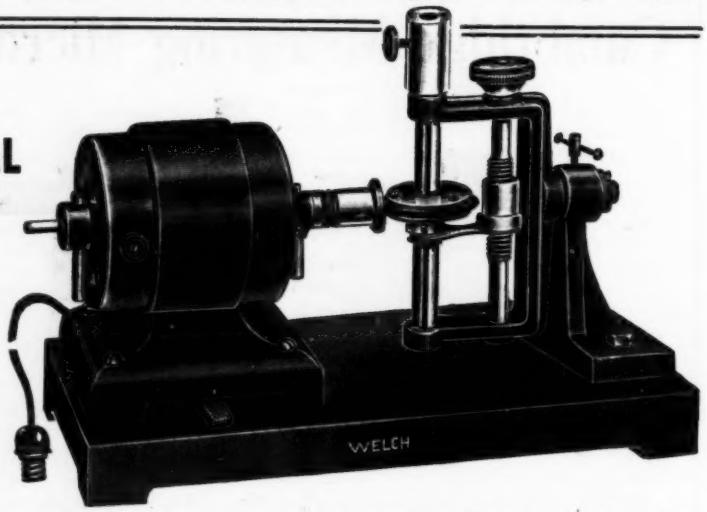

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PREPARATION OF SYNOPSES

Revised, May, 1920

Every article in THE PHYSICAL REVIEW is to be preceded by a synopsis prepared by the author and submitted by him with the manuscript. The synopsis is intended to serve as an aid to the reader by furnishing an index and brief summary or preliminary survey of the contents of the article; it should also be suitable for reprinting in an abstract journal so as to make a reabstracting of the article unnecessary. *The synopsis should, therefore, summarize the information completely and precisely.* Furthermore, in order to enable a reader to tell at a glance what the article is about and to enable an efficient index of its subject matter to be readily prepared, *the synopsis should contain a set of subtitles which together form a complete and precise index of the information contained in the article.* This requires at least one and often several subtitles even for a short synopsis.

In the preparation of synopses, authors should be guided by the following rules, which are illustrated by the synopses in the PHYSICAL REVIEW for January, February and March, 1920.* The new information contained in an article should first be determined by a careful analysis; then the subtitles should be formulated; and finally the text should be written and checked.

Rules

1. *Material not new* need not be analyzed or described; a valuable summary of previous work, however, should be noted.

2. *The subtitles should together include all the new information;* that is, every measurement, observation, method, improvement of apparatus, suggestion and theory which is presented by the author as new and of value in itself.

3. *Each subtitle should describe the corresponding information so precisely* that the chance of any investigator being misled into thinking the article contains the particular information he desires when it does not, or *vice-versa,* may be small. "X-ray patterns of metals" is too broad unless all metals have been studied, for an investigator may be interested, at the time, in only one metal; but "X-ray patterns of aluminum, effect of rolling" evidently satisfies this rule. It is particularly desirable that ranges of variation of temperature, wave-length, pressure, etc., be given in the subtitle.

General subtitles, such as "Object" or "Results," should not be employed, as they do not help to describe the specific information given in the article.

4. *The text should summarize the author's conclusions and should transcribe numerical results of general interest,* including those that might be looked for in a table of physical and chemical constants, with an indication of the accuracy of each. It should give all the information that anyone, not a specialist in the particular field involved, might care to have in his note book.

5. *The text should be divided into as many paragraphs* as there are distinct subjects concerning which information is given. Parts of subtitles may be scattered through the text but the subject of each paragraph, however short, must be indicated at the beginning.

6. *Complete sentences* should be used except in the case of subtitles. The synopsis should be made as readable as the necessary brevity will permit.

*The rules and illustrative synopses were prepared by G. S. Fulcher of the National Research Council.

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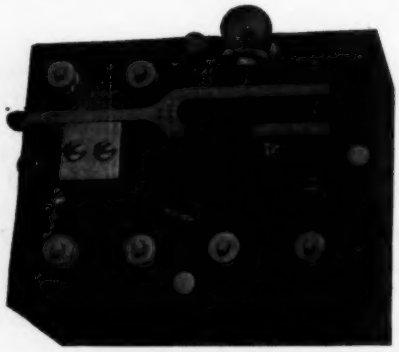

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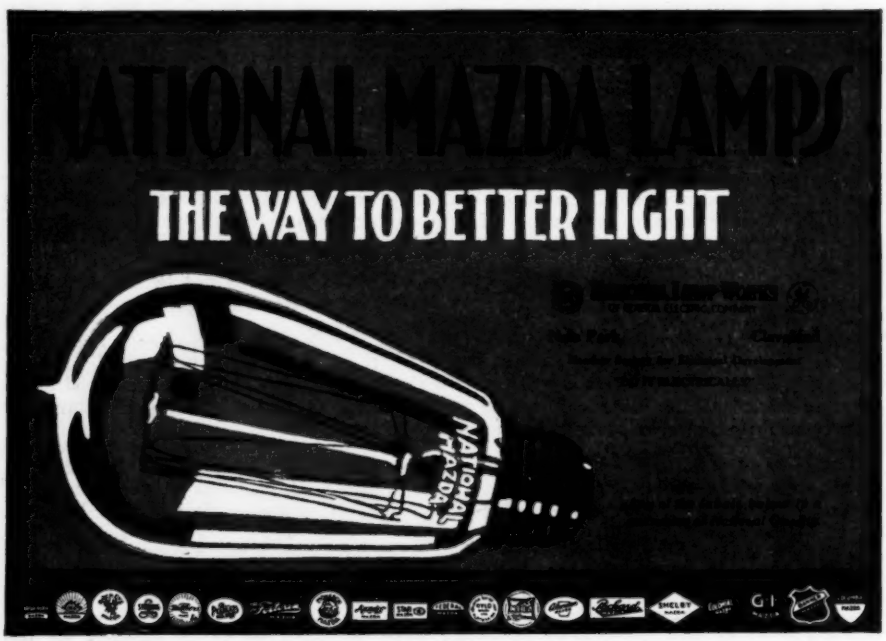

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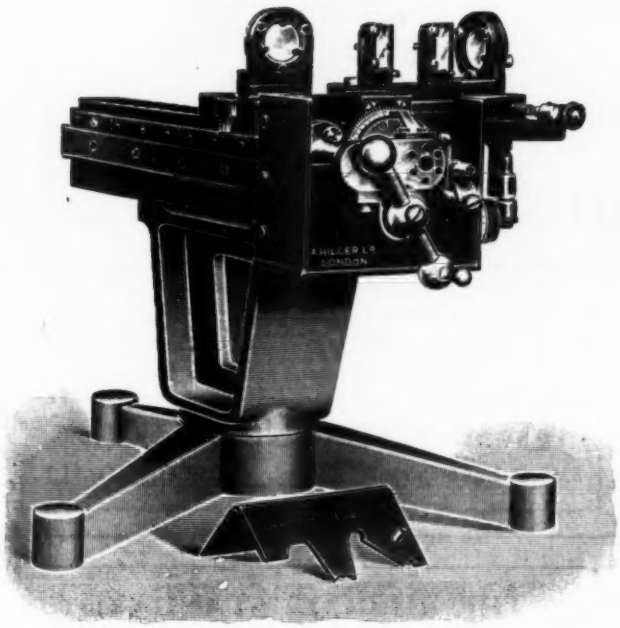

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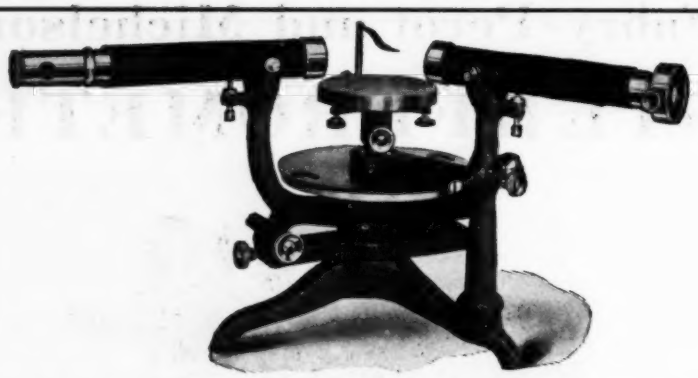

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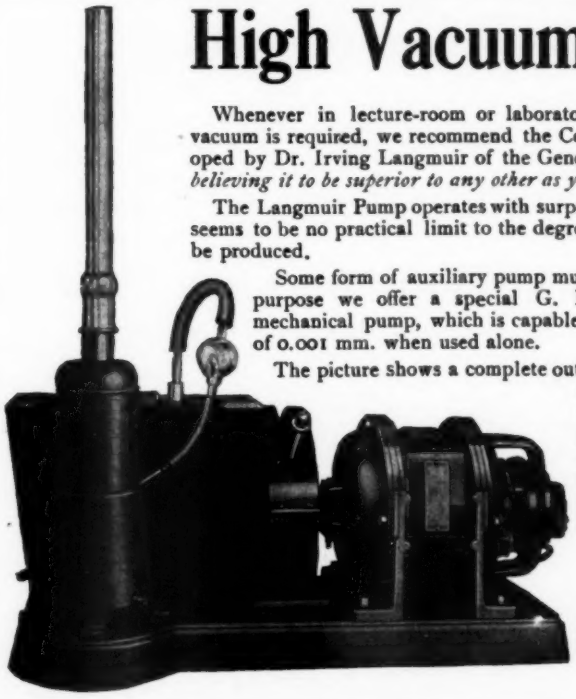

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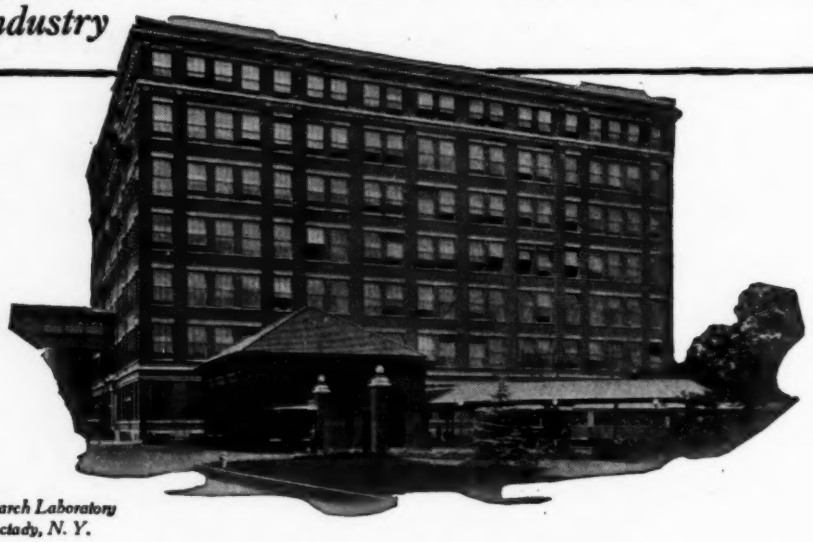

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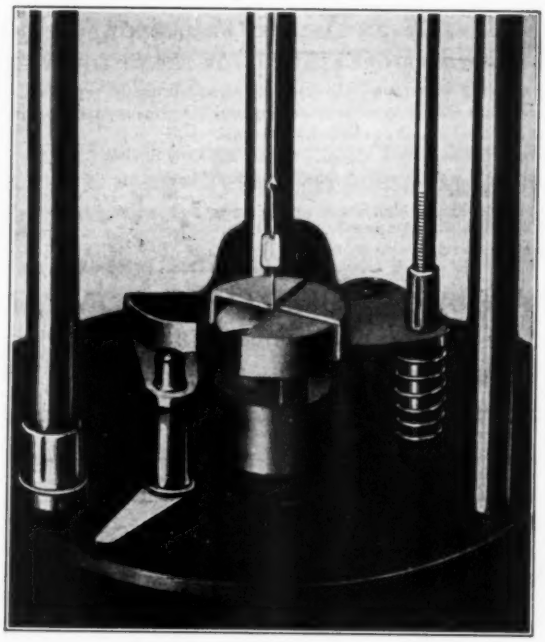

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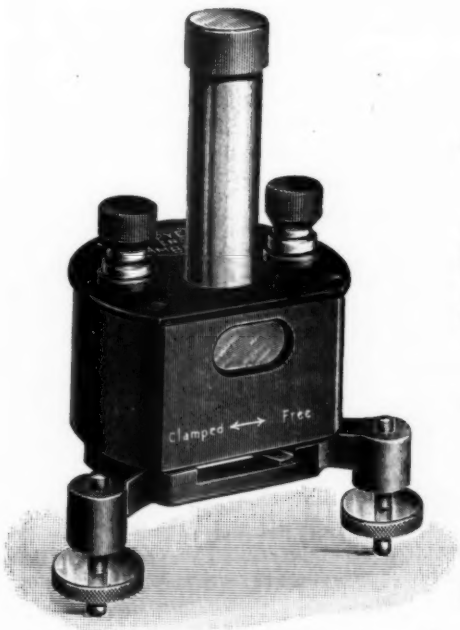

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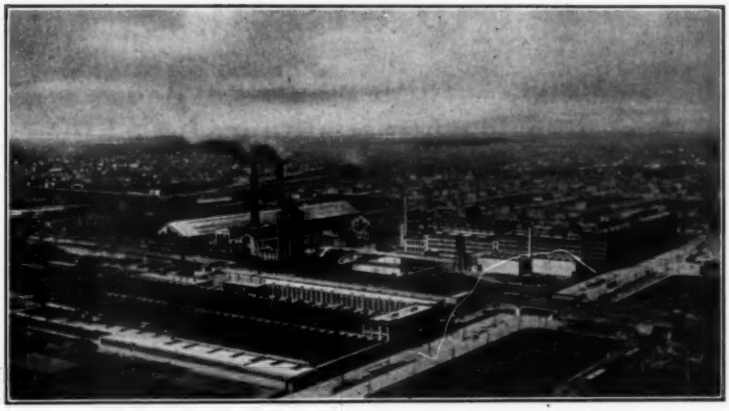

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A BRIEF SURVEY

of the History of

Falling Body Experiments

No. F1191.

Reveals the fact that the goal of each contributor to the long series of experiments since the time of Galileo has been

(1) to approximate as nearly as possible the condition of a freely falling body;

(2) to eliminate any influence other than gravity, such as mechanical friction and resistance of the air;

(3) to divide the time of fall into small, accurately measurable intervals; and

(4) to obtain an exact, readily measured distance of fall.

Galileo confined his quantitative efforts largely to the inclined plane. Atwood, over 100 years ago, improved the situation by obtaining approximately the condition of a freely falling body with the Atwood's Machine which, however, failed to eliminate friction and incorporated into the calculation the moment of inertia of the pulley.

Morin's Apparatus, designed after a suggestion of General Poncelet, introduced a new idea—that of a falling object which traced its own path on a screen moving at uniform speed in a horizontal direction. A later method, and one which has met with the greatest success, is that of a falling tuning fork which traces a sinuous path on a smoked glass plate. The difficulty with both methods is the impossibility of eliminating friction between the marking stylus and the plate, between the guide rods and the falling body, and between the bulky falling body and the air.

Whiting combined the pendulum as a time marker with a freely falling ball of small dimensions and established a method which, while possessing possibilities, was difficult to reduce to one of any high degree of precision.

The most accurate methods of determining the value of *g* are the Chronograph and the Pendulum methods, including the Coincidence Pendulum and Kater's Reversible Pendulum. The Chronograph method, while capable of a high degree of accuracy, is poorly adapted for student laboratory work. The pendulum methods make possible a high degree of accuracy but possess the pedagogical disadvantage that no freely falling body is employed.

The apparatus illustrated here was adapted from earlier forms by Prof. Carman and his associates at the University of Illinois in an attempt to furnish the student with a method which employs a freely falling body and an accurate but simple timing device, with which accurate results might be obtained. As will be noted from the illustration, this modification possesses a number of refinements which contribute materially to the ease and accuracy of the experiment.

The time is obtained from a pendulum of special shape mounted on a steel rod about 150 cm. high, which is supported by a substantial tripod. The pendulum bob is a rectangular block supported by two rods, which are attached at their upper ends to a knife edge yoke. A ball is held by an electromagnet adjustably suspended above the "at rest" position of the center of the flat pendulum bob. A second electromagnet in series holds the pendulum in its position of maximum displacement. When the circuit is broken, the ball and the pendulum are released simultaneously. By adjusting either the height of fall, or the pendulum period the ball may be made to strike the bob at its middle point. The point at which the ball strikes is registered by a piece of carbon paper on a white paper fastened to the bob. The period of the pendulum is readily adjusted by means of a sliding weight. When the ball strikes the pendulum at its middle point, the time of fall is equal to the pendulum's quarter period, which may be accurately obtained. Then having measured the distance of fall, "g" as obtained from the formula $h = \frac{1}{2}gt^2$. A series of experiments made by an experienced investigator in our own laboratory with such an apparatus resulted in an average error of not more than one-half per cent.

F1191. ACCELERATION APPARATUS, Pendulum Form, (See S. S. & M., June, 1915, p. 469), complete with electric wiring, but without batteries.....$45.00

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